KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education

121/1

Paper 1

Serial No.
12112024

MATHEMATICS Alt. A

Nov. 2024 – 2½ hours

Name: ……..... Index Number: ……...…...……...…....

Candidate's signature: …….. Date: ……..……..

Instructions to candidates

(a) Write your name and index number and class in the spaces provided above.

(b) Sign and write the date of examination in the spaces provided above.

(c) The paper consists of **two** sections. **Section I** and **Section II.**

(d) Answer **ALL** the questions in Section I and any **FIVE** questions in Section II.

(e) **Show all the steps in your calculations, giving your answer at each stage in the spaces provided below each question.**

(f) Marks may be given for correct working even if the answer is wrong.

(g) **Non-programmable** silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.

(h) ***This paper consists of 18 printed pages.***

(i) ***Candidates must check the question paper to ascertain that all pages are printed as indicated and that no questions are missing***

(j) **Candidates should answer the questions in English.**

For Examiner's Use

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**Total**

SECTION II

17	18	19	20	21	22	23	24	**Total**

Grand Total

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SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1. Without using mathematical tables or a calculator, evaluate $\frac{0.13\times 0.3-0.003}{0.09}$ giving the answer in decimal form. (3 marks)

2. Simplify the expression $\frac{4x^2-9}{2x^2+x-6}$. (3 marks)

3. A straight line L_1 whose equation is $y = 2 - \frac{1}{3}x$ meets the y – axis at Q. Another straight line L_2 is perpendicular to L_1 at Q. Find the equation of L_2 in the form $y = mx + c$ where m and c are constants. (3 marks)

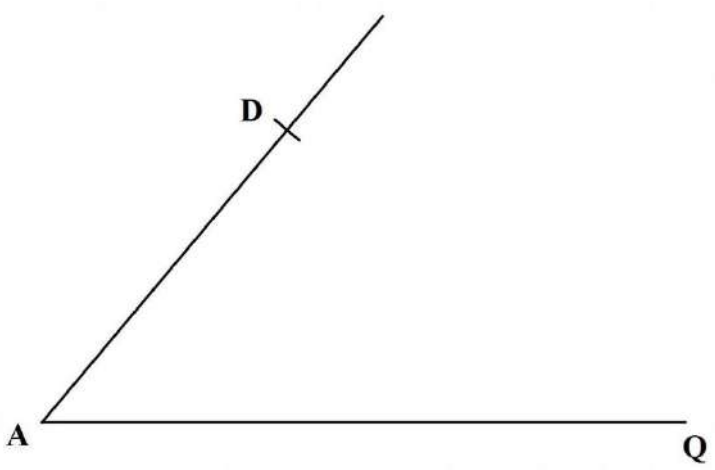
4. A circle of radius 3 cm passes through all the vertices of a regular hexagon. Determine the area of the circle that lies outside the hexagon. (3 marks)

5. Solve for x in the equation $25^{x} = 125^{\frac{2}{3}} \div 5^{-1}$. (3 marks)

6. A carpenter had two big pieces of wood of equal length. The carpenter cut the first piece into smaller pieces of length 15 cm each without remainder. The carpenter cut the second piece into smaller pieces of length 24 cm each without remainder. Determine the minimum length of each of the big pieces of wood. (2 marks)

7. A cylindrical container of internal radius 10.5 cm has a hemispherical base. The container has water up to a height of 30.5 cm. Calculate the surface area of the container that is in contact with water. $\left(\text{Take } \pi = \frac{22}{7}\right)$ (4 marks)

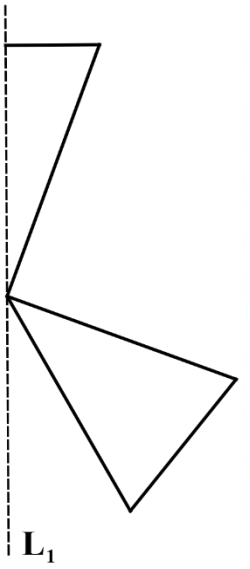
8. In the following figure, points A and D are vertices of a trapezium ABCD. Line DC is parallel to line AQ. Line DC = 4 cm. Point B lies on the line AQ such that angle DCB = 90°.



Using a rule and a pair of compasses only, complete the trapezium. Hence, measure the length of line AB. (4 marks)

9. The length of a minor arc AB of a circle centre O is 10 cm. The arc AB subtends an angle of 1.25 radians at O. Calculate the area of the minor sector AOB. (3 marks)

10. The following figure represents part of a pattern. A line of symmetry, L_1, of the pattern is also shown.



Complete the pattern and hence state the order of rotational symmetry of the patterns. (3 marks)

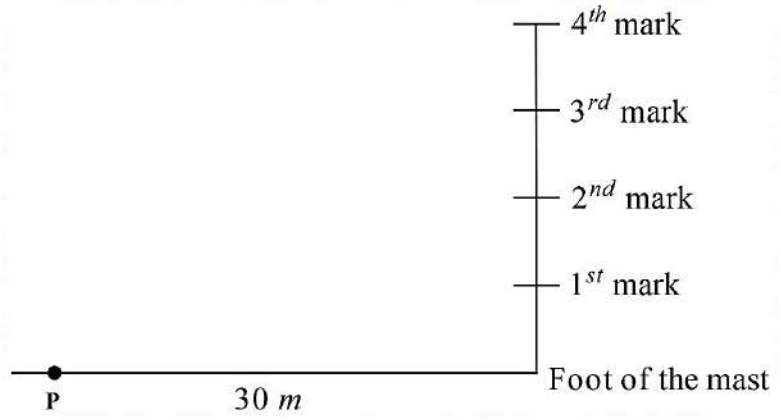
11. A Kenyan bank bought and sold foreign currencies on two different days as shown.

		Buying (In Ksh)	**Selling (In Ksh)**
12/1/2021	1 South African Rand	7.08	7.10
12/2/2021	100 Tanzanian Shillings	21.12	21.25

A South African tourist arrived in Kenya on 12/1/2021 with 15 000 South African rands. He changed the whole amount to Kenyan shillings. While in Kenya, he spent a total of Ksh. 53 075 and changed the balance to Tanzanian shillings before leaving for Tanzania on 12/2/2021. Determine, in Tanzanian shillings, the amount he received. (3 marks)

12. The coordinates of points O, A, B and C are $(0,0),(2,1),(4,4)$ and $(-1,7)$ respectively. The coordinates of point D is (x,y). Point M is the midpoint of line CD and it satisfies the vector equation $\mathbf{OM} = \mathbf{OA} + \frac{5}{2}\mathbf{AB}$. Determine the coordinates of point D. (3 marks)

13. The following figure (not drawn to scale) represents a communication mast. The mast has been divided into 4 equal parts. A point P is 30 m from the foot of the mast on the same ground level.



The angle of elevation of the 3rd mark from P is 50°. Calculate the height of the mast. (3 marks)

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14. The following frequency distribution table shows the mass in kilograms, of maize flour used by 30 traders.

Mass (in kg)	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of traders	3	8	10	7	2

Calculate the median mass of maize flour sold. (3 marks)

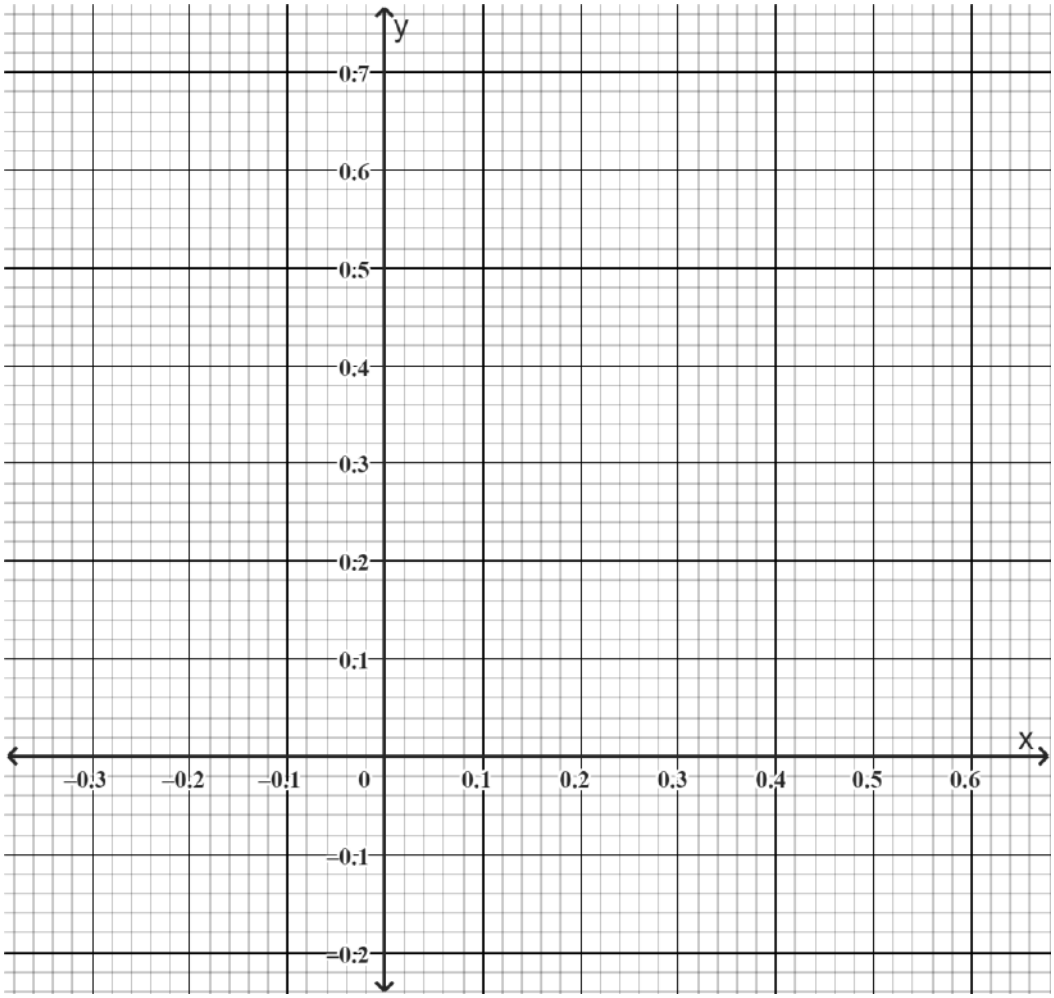
15. The table below shows values of x and some values of y for the curve $y = x^2$ for $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3
y	0	0.25	1		4		9

(a) Complete the table by filling in the missing values of y. (1 mark)

(b) Use the mid ordinate rule with 3 strips to estimate the area bounded by the curve $y = x^2$, the x-axis and the line $x = 3$. (2 marks)

16. Use the cartesian plane provided to solve graphically the simultaneous equations $2x+3y=1.2$ and $5x+4y=1$.



(3 marks)

SECTION II (50 marks)

*Answer only **five** questions in this section in the spaces provided.*

17. On a market day, Abdul has 32 goats while Chebet has 56 goats. Chebet sold twice as many goats as Abdul. After the sale, the number of goats that Abdul and Chebet remained with was in the ratio $3:5$ respectively.

(a) Determine the total number of goats sold by Abdul and Chebet in the market. (4 marks)

(b) Abdul and Chebet raised a total of Ksh. 97 600 from the sale of goats at the market. Abdul's selling price per goat was 5% higher than that of Chebet. Determine the ratio of the earnings of Abdul to Chebet from the sale of goats. (4 marks)

(c) Abdul decreased the selling price per goat in the ratio $19:21$ for the goats that remained. Determine the new selling price per goat. (2 marks)

18. The price of pens in a bookshop changed in the months of February, March and April. The price of a pen was Ksh. 2 less in the month of February that the price of a pen in the month of March. In the month of April the price of a pen was Ksh. 2 more than the price of a pen in the month of March. The bookshop sold pens worth Ksh. 4 200 in February. In April, pens worth Ksh. 4 500 were sold.
Take Ksh. x to be the price of a pen in March.

(a) Write an expression in x for the number of pens sold by the bookshop in:
(i) February; (1 mark)

(ii) April. (1 mark)

(b) The bookshop sold 50 more pens in February than in April. Determine the number of pens sold in February. (6 marks)

(c) Determine the percentage change in the number of pens sold by the bookshop in April compared to the number sold in February. (2 marks)

19. Airport S is 1700 km on a bearing of 300° from airport R. Airport Q is 800 km on a bearing of 215° from R. In the following figure, airport R is represented by point R.



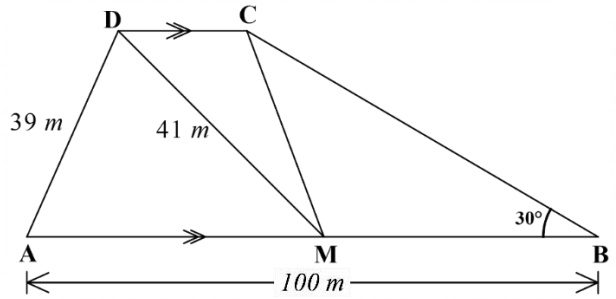
(a) Using a scale of 1 cm to represent 200 km, show on the figure the relative positions of airports S and Q. (3 marks)

(b) Use the scale drawing to determine:
(i) the distance from airport S to airport Q in kilometres; (1 mark)

(ii) the bearing of S from Q. (2 marks)

(c) An aircraft flying at a speed of 400 km/h left S for Q at 8.00 am. Determine the time when the aircraft was exactly 1 000 km from airport R on its way to Q. (4 marks)

20. The following figure represents a piece of land in the shape of a trapezium ABCD. Lines AB and DC are parallel. Point M is the midpoint of AB. The land is divided into three triangular plots. Line AB = 100 m, AD = 39 m, MD = 41 m and angle ABC = 30°.



(a) Calculate the area of the triangular plot AMD and hence the perpendicular distance between the two parallel sides. (4 marks)

(b) Calculate the length of:

(i) BC; (2 marks)

(ii) MC. (2 marks)

(c) Calculate the size of the obtuse angle BMC. (2 marks)

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21. Two matrices **P** and **Q** are such that $\mathbf{P} = \begin{pmatrix} 3 & 7 \\ h & 4 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix}$. The determinant of **PQ** = 3.

(a)

(i) Determine the value of h. (3 marks)

(ii) Find $\mathbf{P}^{-1}$, the inverse of matrix **P**. (2 marks)

(b) Patel and Lagat purchased watches at Ksh. m per watch and Ksh. n per phone. Patel purchased 12 watches and 28 phones for Ksh. 24 600. Lagat purchased 15 watches and 40 phones for Ksh. 34 500.

(i) Form two equations in m and n. (2 marks)

(ii) Use the matrix method to determine the price of a watch and that of a phone. (3 marks)

22. Members of the Environmental Club planted trees. The number of trees planted by each member is recorded below.

24	7	11	6	12	27	8	9	33	23
12	32	19	33	16	21	14	16	17	10
21	18	16	9	17	8	28	13	8	19
9	17	24	18	29	15	12	31	22	14

(a)

(i) Using a class width of 5 and starting with the class 6 – 10, make a frequency distribution table for the data. (2 marks)

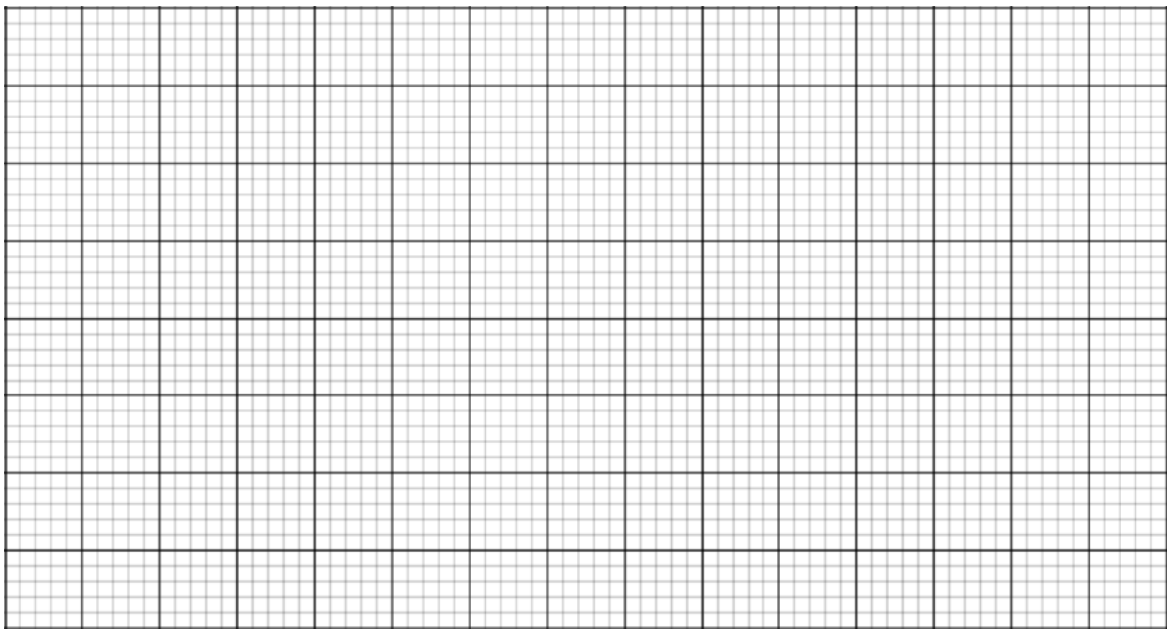
(ii) State the modal class. (1 mark)

(iii) Estimate the mean number of trees planted. (3 marks)

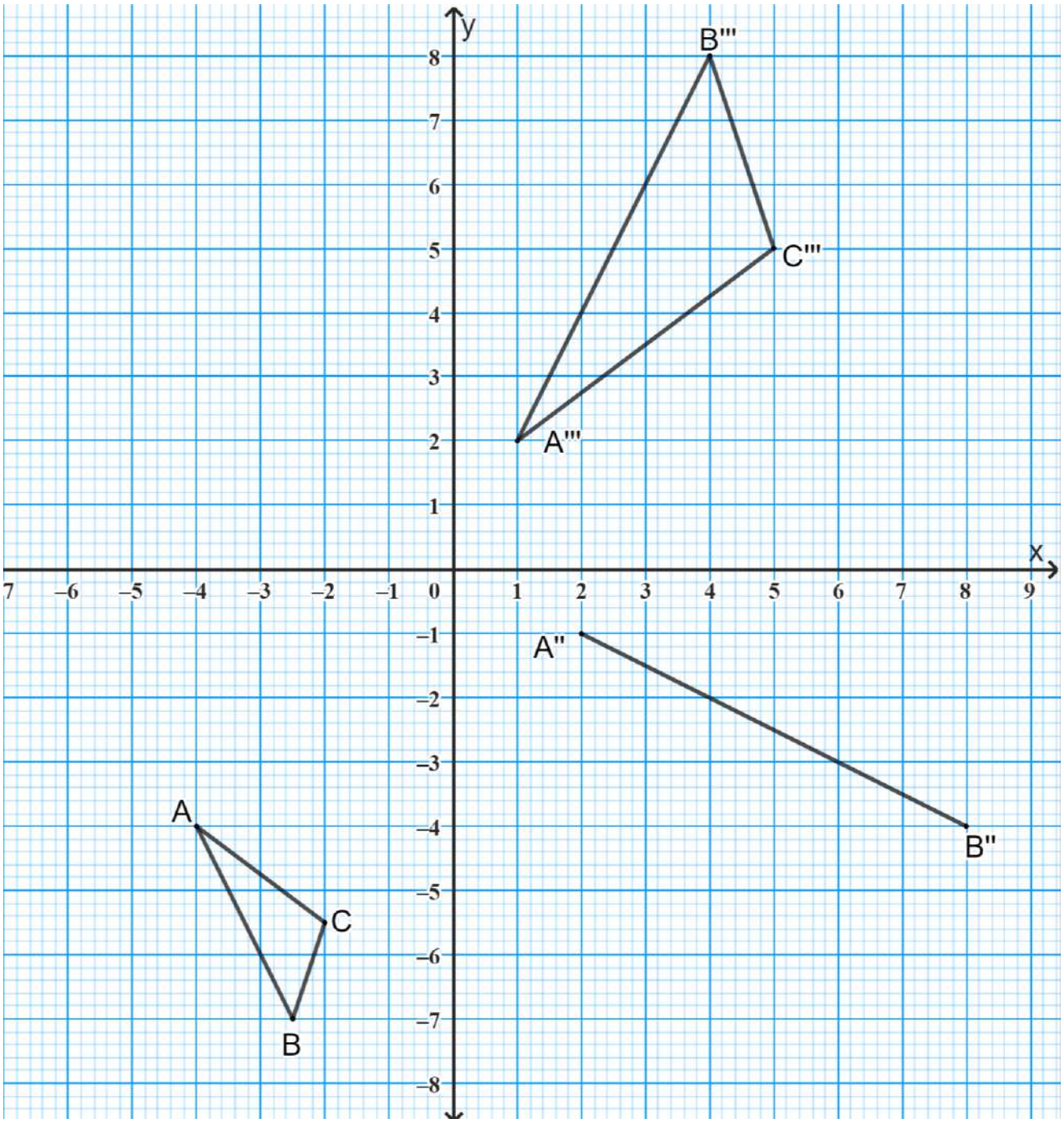
(b) On the grid provided and on the same axes represent the data using:

(i) a histogram; (3 marks)

(ii) a frequency polygon. (1 mark)



23. On the cartesian plane below, triangle ABC has vertices $A(-4,-4)$, $B(-2.5,7)$ and $C(-2,-5.5)$ while triangle $A'''B'''C'''$ has vertices $A'''(1,2)$, $B'''(4,8)$ and $C'''(5,5)$. The line joining $A''(2,-1)$, $B''(8,-4)$ is part of another triangle $A''B''C''$.



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(a) Triangle $A'B'C'$ is the image of triangle ABC under an enlargement centre $(-3,-2)$ and scale factor (S.F.) $= -2$. On the same grid, draw $\Delta A'B'C'$.

(3 marks)

(b) Triangle $A''B''C''$ is the image of triangle $A'B'C'$ under rotation centre $O(0,0)$.

(i) State the angle of rotation. (1 mark)

(ii) Complete triangle $A''B''C''$. (2 marks)

(c) Triangle $A''B''C''$ is the image of triangle $A'B'C'$ under a reflection.

(i) Draw the mirror line. (1 mark)

(ii) Determine the equation of the mirror line in the form $y = mx + c$.

(2 marks)

(d) Describe fully a single transformation that maps triangle $A'B'C'$ onto triangle $A''B''C''$. (1 mark)

24. The gradient of the curve $y = x^3 + 5x^2 + \mathrm{P}x - 18$ at $x = -1$ is -15.

(a) Find:

(i) the value of P; (3 marks)

(ii) the equation of the normal to the tangent to the curve at $x = -1$. (3 marks)

(b) Find the coordinates of the turning points of the curve. (4 marks)

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SECTION I (50 marks)

1.

$$\frac{0.039-0.003}{0.09}=\frac{0.036}{0.09}\times\frac{1000}{1000}$$

$$=\frac{\cancel{36}^{4}}{\cancel{9}_{1}\times 10}=\frac{4}{10}$$

$$=0.4$$

2.

$$\frac{4x^2-9}{2x^2+x-6}=\frac{(2x)^2-3^2}{2x^2+4x-3x-6}$$

$$=\frac{(2x+3)(2x-3)}{2x(x+2)-3(x+2)}$$

$$=\frac{\cancel{(2x-3)}^{1}(2x-3)}{\cancel{(2x-3)}^{1}(x+2)}$$

$$=\frac{2x-3}{x+2}$$

3.

Given $y=-\frac{1}{3}x+2,\ \Rightarrow\ \mathrm{Q}(0,2).$

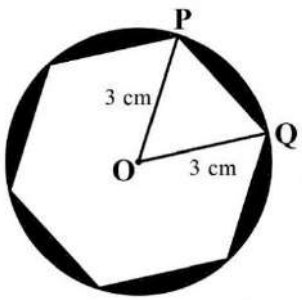
$m_2=-\left(-\frac{1}{3}\right)^{-1}=-1\times\frac{-3}{1}=3$

$y=3x+c$

At Q, $2=3(0)+c$ hence $c=2$.

Equation of L_2 is $\boxed{y=3x+2}$.

4.



$\angle POQ=\frac{360°}{6}=60°$

Area of hexagon

$6\times\frac{1}{2}\times 3^2\times\sin 60°=23.3827\text{cm}^2$

Area of circle $=\frac{22}{7}\times 3^2=28.2857\text{cm}^2$

Shaded area $=28.2857-23.3827$

$=4.903\text{ cm}^2$

5.

$25=5^2,\quad 125=5^3$

$\therefore 5^{2x}=5^{\left(\cancel{3}_1\times\frac{2}{\cancel{3}_1}\right)-(-1)}$

$5^{2x}=5^3\Rightarrow 2x=3$

$x=\frac{3}{2}=1\frac{1}{2}$ or 1.5

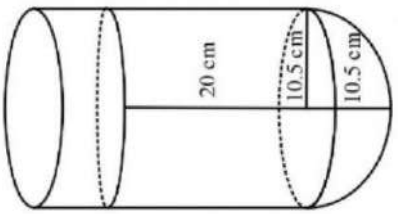
6.

$15=3^1\times 5^1,\ \ 24=2^3\times 3^1$

Minimum length = LCM of 15 and 24

LCM $=2^3\times 3^1\times 5^1=120$ cm

7.

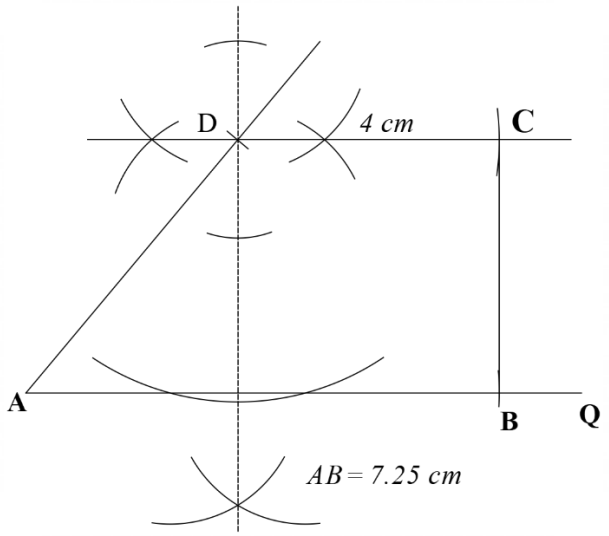


$\text{SA}=\left(2\times\frac{22}{7}\times 10.5^2\right)+\left(2\times\frac{22}{7}\times 10.5\times 20\right)$

$=693+1320$

$=2013\text{ cm}^2$

8.



9.

$\boxed{2\times\frac{22}{7}\times r\times\frac{\theta^c}{2\pi^c}=l}$

$2\times\frac{22}{7}\times r\times\frac{1.25}{2\pi}=10$

$\frac{44}{7}\times r\times 1.25\times\frac{7}{44}=10$

$1.25r=10$

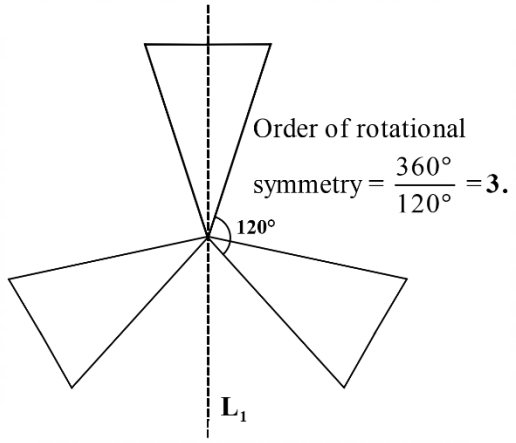
$\therefore r=\frac{10}{1.25}=8\text{ cm}$

$\boxed{\frac{22}{7}\times r^2\times\frac{\theta^c}{2\pi^c}=Area}$

$\text{Area}=\frac{22}{7}\times 8^2\times 1.25\times\frac{7}{44}$

$=40\text{ cm}^2$

10.



11.

The bank buys 1 S.A. rand at Ksh. 7.08

$\therefore 7.08 \times 15\ 000 = \text{Ksh.}106\ 200$

$\text{Remainder} = \text{Ksh.}(106\ 200 - 53\ 075)$

$= \text{Ksh. } 53\ 125$

The bank sells Tsh. 100 for Ksh. 21.25

$\therefore \frac{53\ 125}{21.25} \times 100 = \text{Tsh. } 250\ 000$

12.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \begin{pmatrix}4\\4\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$

$\mathbf{OM} = \begin{pmatrix}2\\1\end{pmatrix} + \frac{5}{2}\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}5\\7.5\end{pmatrix} = \begin{pmatrix}7\\8.5\end{pmatrix}$

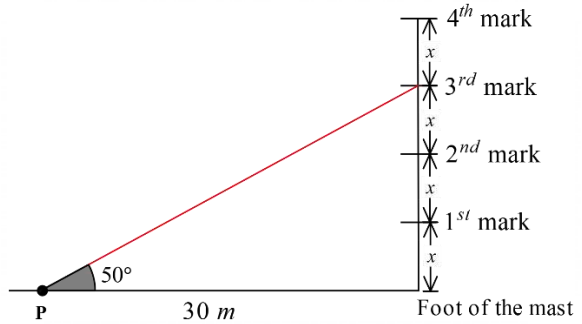
$\therefore$ M $(7, 8.5)$

$\frac{x-1}{2} = 7 \Rightarrow x - 1 = 14$ hence $x = 15$

$\frac{y+7}{2} = 8.5 \Rightarrow y + 7 = 17$ hence $y = 10$

$\therefore \text{D}(15, 10)$

13.



$\tan 50° = \frac{3x}{30}$

$3x = 30 \tan 50°$

$x = \frac{30 \times 1.1918}{3} = 11.918\text{m}$

$\text{Height} = 4x$

$= 11.918 \times 4$

$= 47.672\text{m}$

14.

cf	3	11	21	28	30

$\text{Median position} = \frac{30+1}{2} = 15.5$

Median class is $30 - 39$.

lcb of median class $= 29.5$

$\text{Median} = 29.5 + \left(\frac{(15.5 - 11)}{10} \times 10\right)$

$= 34$

15.

x	0	0.5	1	1.5	2	2.5	3
y	0	0.25	1	**2.25**	4	**6.25**	9

$h = \frac{3}{3} = 1$

$Area = h(y_1 + y_2 + y_3)$

$Area = 1(0.25 + 2.25 + 6.25)$

$= 8.75$ sq. units

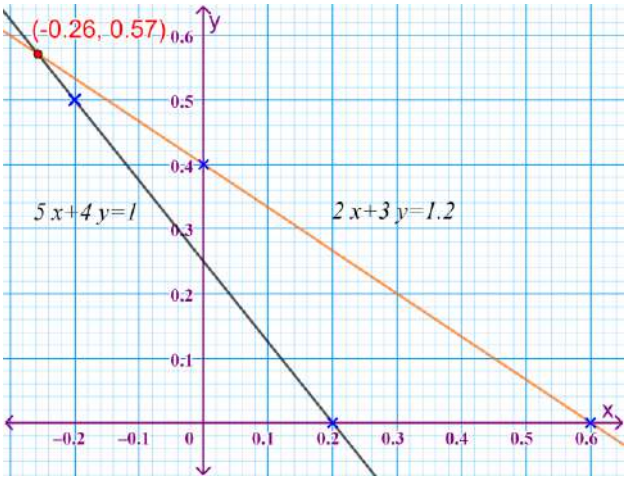
16.

$2x + 3y = 1.2$

x	0	0.6
y	0.4	0

$5x + 4y = 1$

x	–0.2	0.2
y	0.5	0



$x = -0.26$

$y = 0.57$

SECTION II (50 marks)

17.

(a)

Farmer	Abdul	Chebet
Sold	x	$2x$
Remaining	$32-x$	$56-2x$
Selling price	$1.05k$	k

$$\frac{32-x}{56-2x}=\frac{3}{5}$$

$$160-5x=168-6x$$

$$x=8$$

Abdul sold 8 goats

Chebet sold $2\times 8=16$ goats

(b)

$$16k+8(1.05k)=97600$$

$$16k+8.4k=97600$$

$$24.4k=97600\Rightarrow k=4000$$

Chebet sold a goat at Ksh. 4 000

Abdul sold a goat at $1.05\times 4\ 000=$ Ksh. 4 200

Abdul's earnings : Chebet's earnings

$$(4200\times 8):(4000\times 16)$$

$$33600:64000$$

$$21:40$$

(c)

$$\frac{19}{21}\times 4200=\text{Ksh. } 3\ 800$$

18.

(a) (i) $\frac{4200}{x-2}$ (ii) $\frac{4500}{x+2}$

(b)

$$\frac{4200}{x-2}-\frac{4500}{x+2}=50$$

$$4200(x+2)-4500(x-2)=50(x^2-4)$$

$$4200x+8400-4500x+9000=50x^2-200$$

$$50x^2+300x-17600=0$$

$$x^2+6x-352=0$$

$$x^2+22x-16x-352=0$$

$$x(x+22)-16(x+22)=0$$

$$(x-16)(x+22)=0$$

$$x-16=0 \text{ or } x+22=0$$

$$x=16 \text{ or } x=-22$$

Taling the positive value, $x=16$ watches

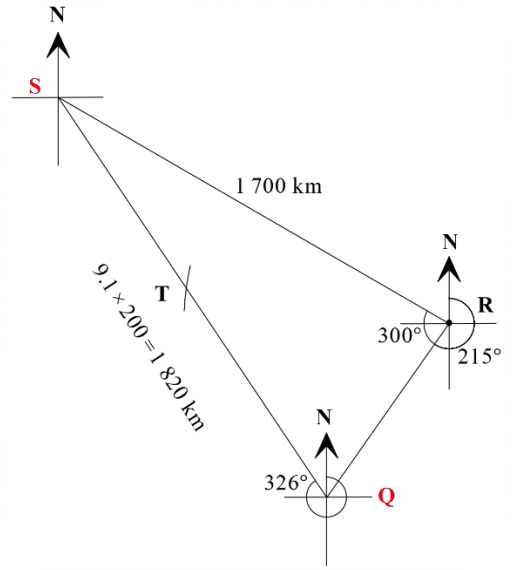
In Feb, $\frac{4200}{16-2}=300$ pens sold.

(c)

$$\%\text{ change}=\frac{50}{300}\times 100=16\frac{2}{3}\%$$

19.

(a)



(b) Mark T; 5 cm (on paper) from R along SQ.

$ST=3.6\times 200=720$ km

Time taken to reach T $=\frac{720}{400}=1\frac{4}{5}h$

$\equiv$ 1hr 48 min

The aircraft will be at T at:

0800 + 1hr 48 min = 0948h//9.48 am.

20

(a) $s=\frac{39+41+50}{2}=65$

Area $=\sqrt{65(65-50)(65-41)(65-39)}$

$=\sqrt{65\times 15\times 24\times 26}=780\text{m}^2$

Let the perpendicular height be *h*.

$$\frac{1}{2}\times 50\times h=780$$

$25h=780\Rightarrow h=31.2\text{m or } 31\frac{1}{5}m$

(b) (i) $\sin 30°=\frac{31.2}{BC}$

$\Rightarrow \text{BC}=\frac{31.2}{\sin 30°}=\frac{31.2}{0.5}=62.4\text{m}$

(ii) $x^2=50^2+62.4^2-(2\times 50\times 62.4\times\cos 30°)$

$x^2=2500+3893.76-5403.84$

$x^2=989.92$

Hence $x=\sqrt{989.92}=31.463m$

(c) $\frac{31.463}{\sin 30°}=\frac{62.4}{\sin y}$

$\sin y=\frac{62.4\sin 30°}{31.463}=\frac{31.2}{31.463}=0.9916$

$y=\sin^{-1}0.9916=82.57°$

Obtuse $\angle\text{BMC}=180°-82.57°$

$=97.43°$

21

(a)

(i) $\mathbf{PQ} = \begin{pmatrix} 3 & 7 \\ h & 4 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} (3\times5)+(7\times3) & (3\times-4)+(7\times-2) \\ (h\times5)+(4\times3) & (h\times-4)+(4\times-2) \end{pmatrix} = \begin{pmatrix} 36 & -26 \\ 5h+12 & -4h-8 \end{pmatrix}$

$\det \mathbf{PQ} = 36(-4h-8)-\left[-26\left((5h+12)\right)\right] = 3$

$\therefore -144h-288-\left[-130h-312\right] = 3$

$-144h-288+130h+312 = 3$

$-14h+24 = 3$

$-14h = -21 \quad \Rightarrow \quad h = 1.5 \text{ or } 1\frac{1}{2}.$

(ii) $\mathbf{P} = \begin{pmatrix} 3 & 7 \\ 1.5 & 4 \end{pmatrix}; \quad \det \mathbf{P} = \begin{vmatrix} 3 & 7 \\ 1.5 & 4 \end{vmatrix} = (3\times4)-(7\times1.5) = 1.5$

$\mathbf{P}^{-1} = \frac{1}{1.5}\begin{pmatrix} 4 & -7 \\ -1.5 & 3 \end{pmatrix} = \begin{pmatrix} \frac{8}{3} & -\frac{14}{3} \\ -1 & 2 \end{pmatrix}$

(b)

(i) $12m+28n = 24600 \quad \Big| \quad 15m+40n = 34500$

$3m+7n = 6150...(i) \quad \Big| \quad 1.5m+4n = 3450...(ii)$

(ii) $\begin{pmatrix} \frac{8}{3} & -\frac{14}{3} \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 7 \\ 1.5 & 4 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} \frac{8}{3} & -\frac{14}{3} \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 6150 \\ 3450 \end{pmatrix}$

$\begin{bmatrix} (\frac{8}{3}\times3)+(-\frac{14}{3}\times1.5) & (\frac{8}{3}\times7)+(-\frac{14}{3}\times4) \\ (-1\times3)+(2\times1.5) & (-1\times7)+(2\times4) \end{bmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{bmatrix} (\frac{8}{3}\times6150)+(-\frac{14}{3}\times3450) \\ (-1\times6500)+(2\times3450) \end{bmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 300 \\ 750 \end{pmatrix}$

$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 300 \\ 750 \end{pmatrix} \Rightarrow m = 300 \text{ and } n = 750 \quad \therefore \text{watch's price} = \text{Ksh. } 300, \text{ phone's price} = \text{Ksh. } 750$

22

(a) (i)

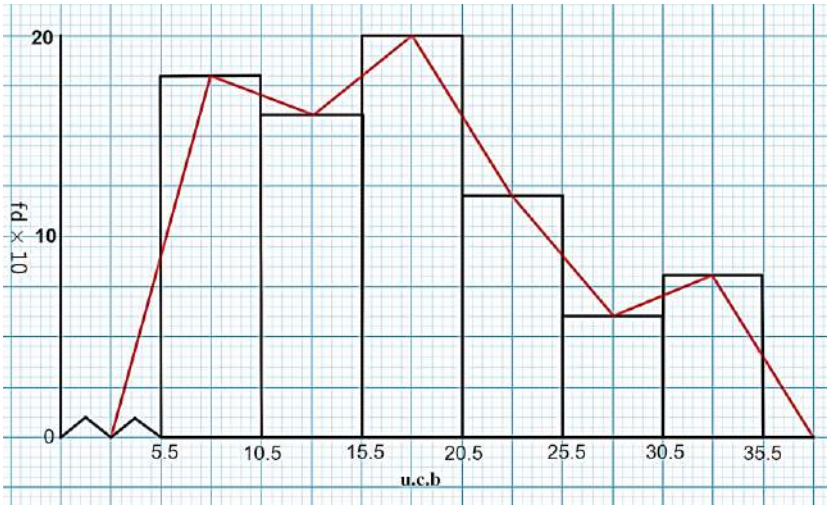
Trees (*class*)	*Number* (*f*)	*Midpoint*, (*x*)	*fx*
6 – 10	9	8	72
11 – 15	8	13	104
16 – 20	10	18	180
21 – 25	6	23	138
26 – 30	3	28	84
31 – 35	4	33	132
	$\sum f = \mathbf{40}$		$\sum fx = \mathbf{710}$

(ii) 16 – 20

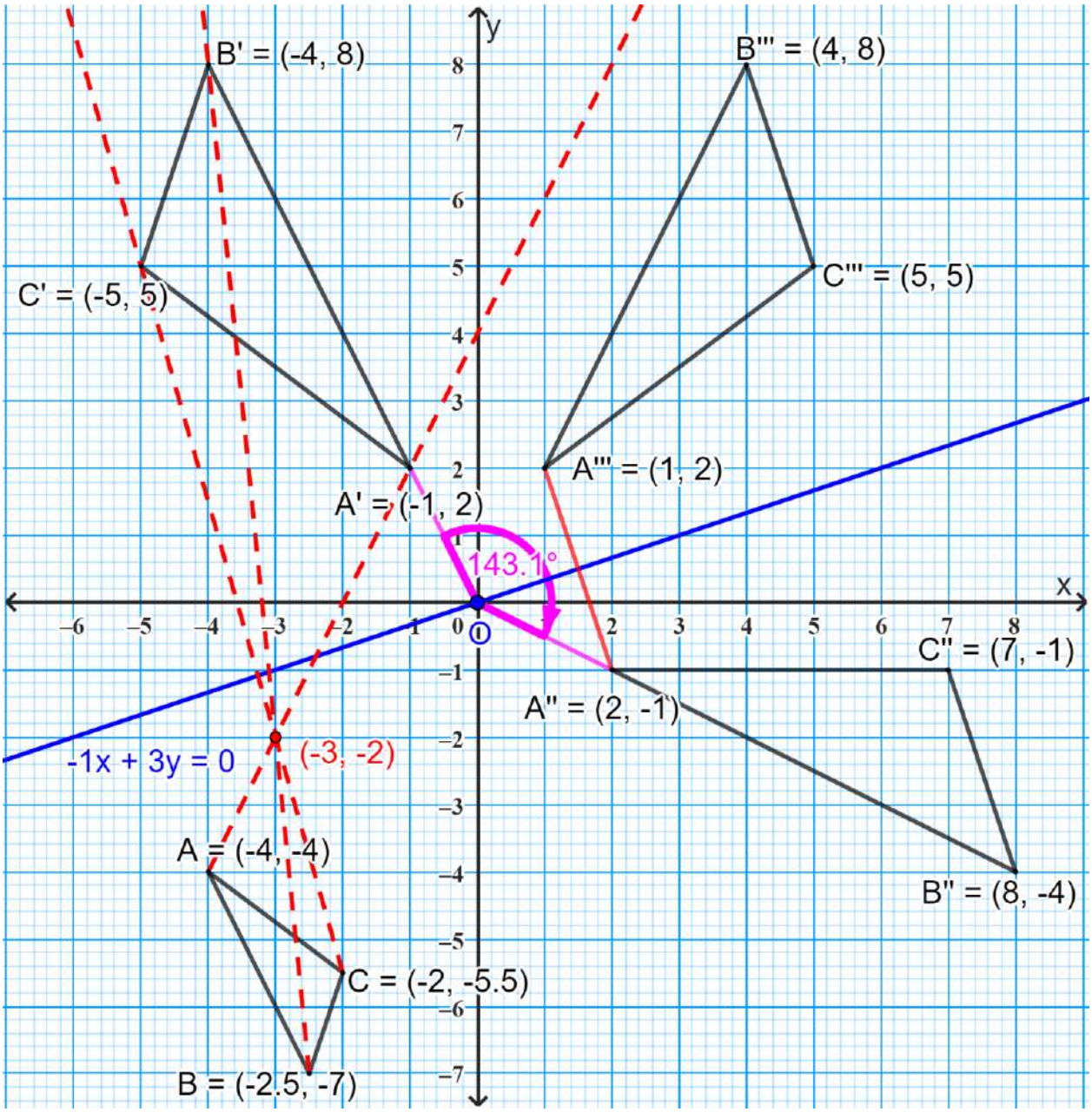
(iii) $\bar{x} = \frac{\sum fx}{\sum f}$

$= \frac{710}{40} = 17.75$

(b)



23



(a) $A'B'C'$ the image of ABC under an enlargement centre $(-3,-2)$ and (S.F.) $=-2$.

(b) $A''B''C''$ the image of $A'B'C'$ under rotation centre $O(0,0)$.

(i) Angle of rotation. **143.1°**

(ii) Completing triangle $A''B''C''$.

(c) $A'''B'''C'''$ the image of triangle $A''B''C''$ under a reflection.

(i) Mirror line **drawn in blue.**

(ii) Equation of the mirror line

Using $(6,2)$ and $(-6,-2)$;

Gradient, $m=\dfrac{2-(-2)}{6-(-6)}=\dfrac{4}{12}=\dfrac{1}{3}$

$y-$intercept, $c=0$

$\therefore$ using $y=mx+c$, $y=mx$ hence the eqn $\boxed{y=\dfrac{1}{3}x}$

(d) $A'B'C'$ is mapped onto $A''B''C''$ by reflection in the *y-axis* (the line $x=0$.)

24

(a)

(i) $\dfrac{dy}{dx} = 3x^2 + 10x + P$

At $x = -1$, $\dfrac{dy}{dx} = 3(-1)^2 + 10(-1) + P = -15$

$3 - 10 + P = -15$

$P = -15 + 10 - 3 = -8$

(ii) Gradient of normal $= \dfrac{-1}{-15} = \dfrac{1}{15}$

At $x = -1$, $y = (-1)^3 + 5(-1)^2 + (-8)(-1) - 18 = -6$

$\dfrac{y+6}{x+1} = \dfrac{1}{15}$

$x + 1 = 15y + 90 \Rightarrow x - 15y - 89 = 0$

(b)

$y = x^3 + 5x^2 - 8x - 18$

$\dfrac{dy}{dx} = 3x^2 + 10x - 8$

At a truning point, $3x^2 + 10x - 8 = 0$

$3x^2 + 12 - 2x - 8 = 0$

$3x(x+4) - 2(x+4) = 0$

$x + 4 = 0$ or $3x - 2 = 0$

$x = -4$ or $x = \dfrac{2}{3}$

When $x = -4$, $y = (-4)^3 + 5(-4)^2 - 8(-4) - 18 = 30$

When $x = \dfrac{2}{3}$, $y = \left(\dfrac{2}{3}\right)^3 + 5\left(\dfrac{2}{3}\right)^2 - 8\left(\dfrac{2}{3}\right) - 18 = -20\dfrac{22}{27}$

$\therefore$ the turning points occur at $(-4, 30)$ and $\left(\dfrac{2}{3}, -20\dfrac{22}{27}\right)$

KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education

121/2

Paper 2

Serial No.
12122024

MATHEMATICS Alt. A

Nov. 2024 – 2½ hours

Name: ……..... Index Number: ……..…..……..…....

Candidate's signature: …….. Date: ……..……..

Instructions to candidates

(a) Write your name and index number and class in the spaces provided above.

(b) Sign and write the date of examination in the spaces provided above.

(c) The paper consists of **two** sections. **Section I** and **Section II.**

(d) Answer **ALL** the questions in Section I and any **FIVE** questions in Section II.

(e) **Show all the steps in your calculations, giving your answer at each stage in the spaces provided below each question.**

(f) Marks may be given for correct working even if the answer is wrong.

(g) **Non-programmable** silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.

(h) ***This paper consists of 19 printed pages.***

(i) ***Candidates must check the question paper to ascertain that all pages are printed as indicated and that no questions are missing***

(j) **Candidates should answer the questions in English.**

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**Total**

SECTION II

17	18	19	20	21	22	23	24	**Total**

Grand Total

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SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1. An arithmetic progression (AP) is given as $600+650+700+750+...$ Determine:
(a) the 30^{th} term of the AP; (2 marks)

(b) the sum of the first 30 terms of the AP. (2 marks)

2. The quadratic equation $5x^2+kx+20=0$ has only one root. Determine the possible values of k. (2 marks)

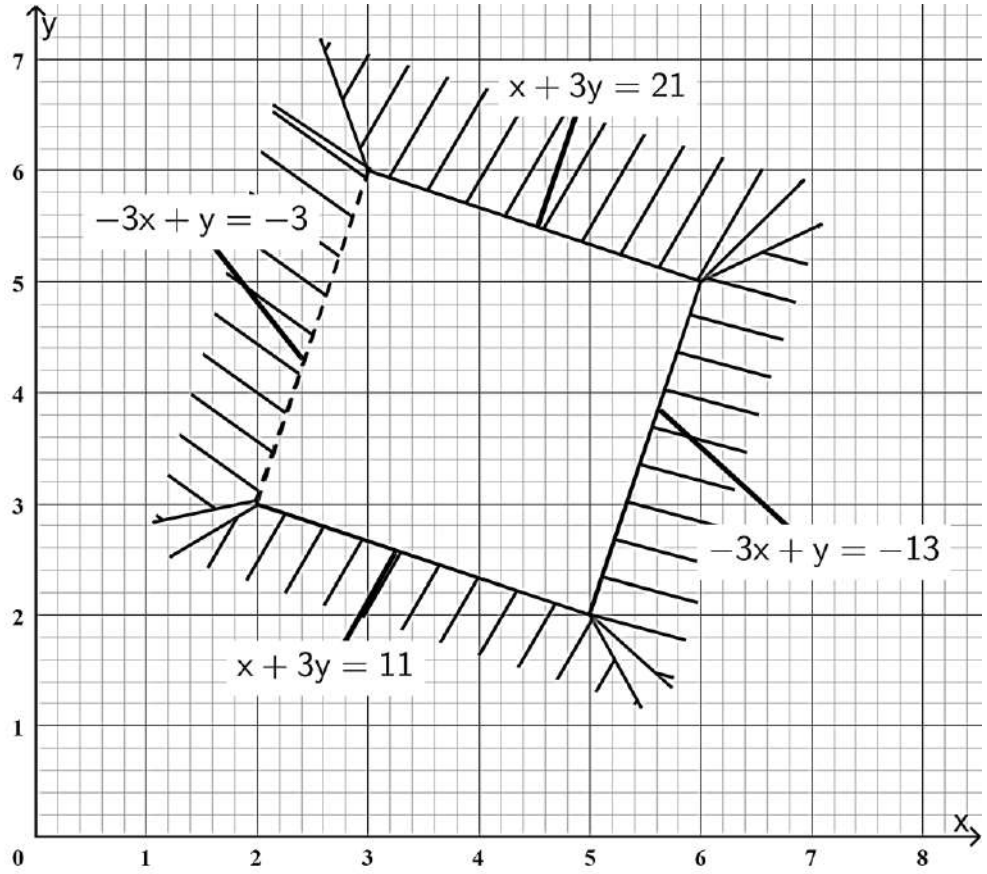
3. Without using mathematical tables or a calculator, evaluate $\dfrac{\log 125+\log 64}{\log \sqrt[6]{5}+\log \sqrt[3]{2}}$.

(3 marks)

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4. Make x the subject of the formula $y = \frac{a}{b^x}$. (3 marks)

5. The unshaded region on the Cartesian plane satisfies the inequalities $x + 3y \leq 21, -3x + y < -3, -3x + y \geq -13$ and $x + 3y \geq 11$.

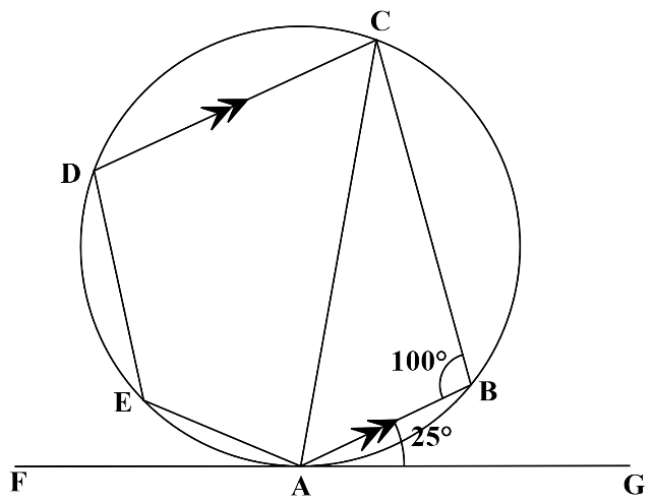


Find the maximum value of $(x + 4y)$ for the integral coordinates $P(x, y)$ lying in the unshaded region. (3 marks)

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6. An aircraft flew due west from point $A(39.64°N, 50°E)$ to $B(39.64°N, 20°W)$.
Calculate the distance covered by the aircraft correct to the nearest km.
$\left(\text{Take } \pi = \frac{22}{7} \text{ and R} = 6370 \text{ km}\right)$ (3 marks)

7. In the following figure; A, B, C, D and E are points on the circumference of the circle. Line AB is parallel to line DC and line FAG is tangent to the circle at A.
Angle GAB = 25° and $\angle ABC = 100°$.



Determine the size of:
(a) $\angle BAC$; (1 mark)

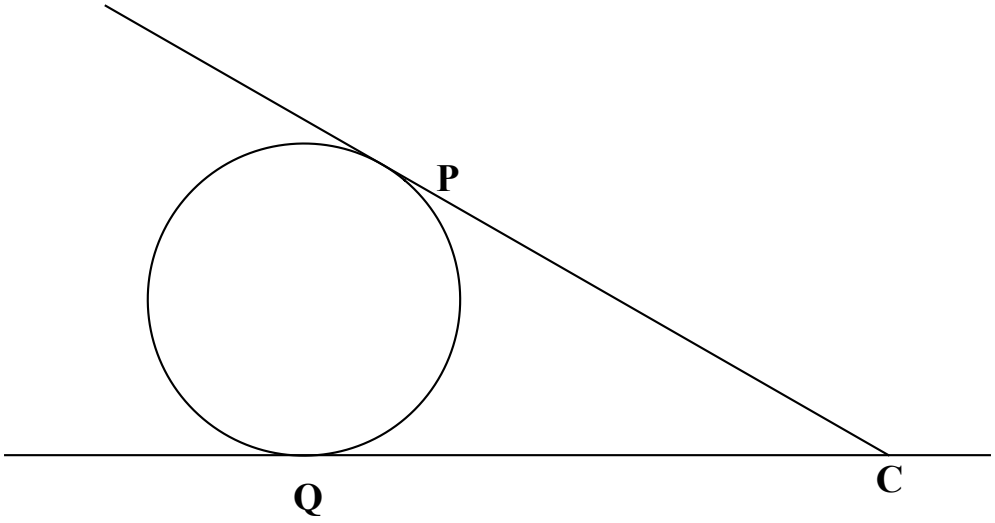
(b) $\angle AED$. (2 marks)

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8. The triangle ABC with vertices $\mathrm{A}(1,0), \mathrm{B}(3,0)$ and $\mathrm{C}(1,2)$ is transformed by the matrix $\mathbf{T} = \begin{pmatrix} 3k & 1.6 \\ 3k & -0.9 \end{pmatrix}$ onto triangle $\mathrm{A'B'C'}$. Given that the area of triangle $\mathrm{A'B'C'}$ is 6 square units, determine the value of *k*. (3 marks)

9. Solve the equation $6\cos^2 x + \sin x = 4$ for $0° \leq x \leq 180°$, giving the answer correct to 2 decimal places. (3 marks)

10. The figure below shows a circle. Lines CP and CQ are tangents to the circle at points P and Q respectively.



The circle is to be inscribed in a triangle ABC. Point B lies on CQ produced and $\angle CBA = 90°$. Use a ruler and a pair of compasses only to:

(a) locate point O, the centre of the circle; (2 marks)

(b) complete triangle ABC. (2 marks)

11. The deviations of the masses of 10 students from an assumed mean are:

$-10,\ -5,\ -2,\ 1,\ 4,\ 5,\ 7,\ 8,\ 9,\ 13$

The mass of the heaviest student was 58 kg. Calculate the mean mass of the students.

(3 marks)

12. The following table shows part of a monthly income tax rates for a certain year.

Monthly taxable income (Ksh.)	Tax rate (%)
0 to 11 180	10
11 181 to 21 714	15
21715 to 32 248	20

In a certain month an employee paid a net tax of Ksh. 2 200 after getting a tax relief of Ksh. 1 280. Calculate the employee's taxable income that month. (3 marks)

13. The equation of a circle is given by $x^2 + y^2 - 3x + 4y = 0$. Determine:

(a) the coordinates of the centre of the circle; (2 marks)

(b) the area of the circle in terms of π. (1 mark)

14. The position vectors of points A, B and C are such that $\mathbf{OA} = 3\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}, \mathbf{OB} = \mathbf{j} + 8\mathbf{k}$ and $\mathbf{OC} = -2\mathbf{i} + 5\mathbf{j} + 16\mathbf{k}$.
Show that the points A, B and C are collinear. (3 marks)

15. A particle starts from point O and moves in a straight line so that its velocity v ms^{-1} after time t seconds is given by $v = 9t^2 - 18t + 10$. Calculate the distance covered by the particle between the time $t = 1$ second and $t = 2$ seconds. (3 marks)

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16. The following table shows the number of units (U) of water consumed by 6 households in a month. The corresponding amount (A) charged is also given.

No. of units (U)	0.8	1.7	2.2	2.9	3.5	4.2
Amount (A) charged in Ksh.	200	340	380	480	600	720

(a) Using the scale 2 cm to represent 1 unit on the *x*-axis and 1 cm to represent Ksh. 100 on the *y*-axis, draw the line of best fit for the data on the grid provided. (3 marks)



(b) Estimate the cost of 1.5 units of water. (1 mark)

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SECTION II (50 marks)

*Answer only **five** questions in this section in the spaces provided.*

17. A poultry dealer has to types of chicken feeds: type A and type B. He sells 1 kg of type A at Ksh. 45 and 1 kg type B at Ksh. 30. He makes a profit of 20% per kg of type A feed sold and 25% per kg of typer B feed sold. He also sales mixtures of type A and type B feeds.

(a) Determine the amount of profit made by the dealer for selling 1 kg of:

(i) type A feed. (1 mark)

(ii) type B feed. (1 mark)

(b) Type A and type B feeds were mixed in the ratio $3:7$. Calculate:

(i) the selling price of 1 kg of the mixture; (2 marks)

(ii) the profit made by the dealer in selling 50 kg of the mixture. (2 marks)

(c) The dealer made a profit of Ksh. 1 387.50 for the sale of 200 kg of a different mixture of type A and type B feeds. Determine the ratio of type A feed to that of type B feed in the mixture. (4 marks)

18.

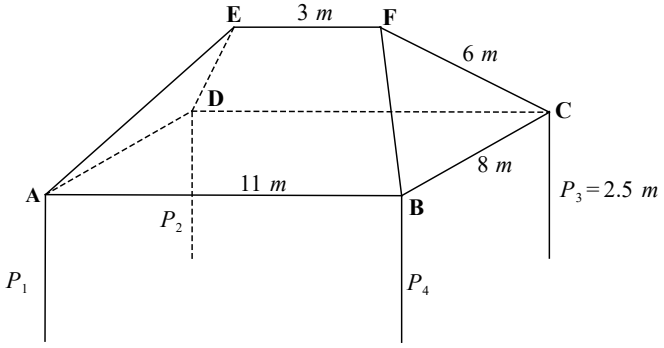
(a) A quantity P is partly constant and partly varies as the square root of a quantity Q. Given that P = 20 when Q = 4 and that P = 60 when Q = 100, find Q when P = 22. (4 marks)

(b) Three quantities, T, U and V are such that T varies directly as the square of $(10-U)$ and inversely as the cube root of V. When T = 12, U = 4 and V = 8.

(i) Determine the equation connecting T, U and V. (3 marks)

(ii) Find U when $T=5\frac{2}{5}$ and $V=15\frac{5}{8}$. (3 marks)

19. The following figure shows a tent erected on a level ground. The roof ABCDEF of the tent is supported by four vertical posts P_1, P_2, P_3 and P_4 each of height 2.5 m. The ridge EF = 3 m is centrally placed.
Further, AB = 11 m, BC = 8 m and FB = FC = ED = EA = 6 m.



Calculate:

(a) the length of the projection of FC on the ground correct to 4 significant figures. (3 marks)

(b) the height of the ridge EF above the ground. (3 marks)

(c) the angle between edge FB and edge FC. (2 marks)

(d) the angle between the plane FBC and the ground. (2 marks)

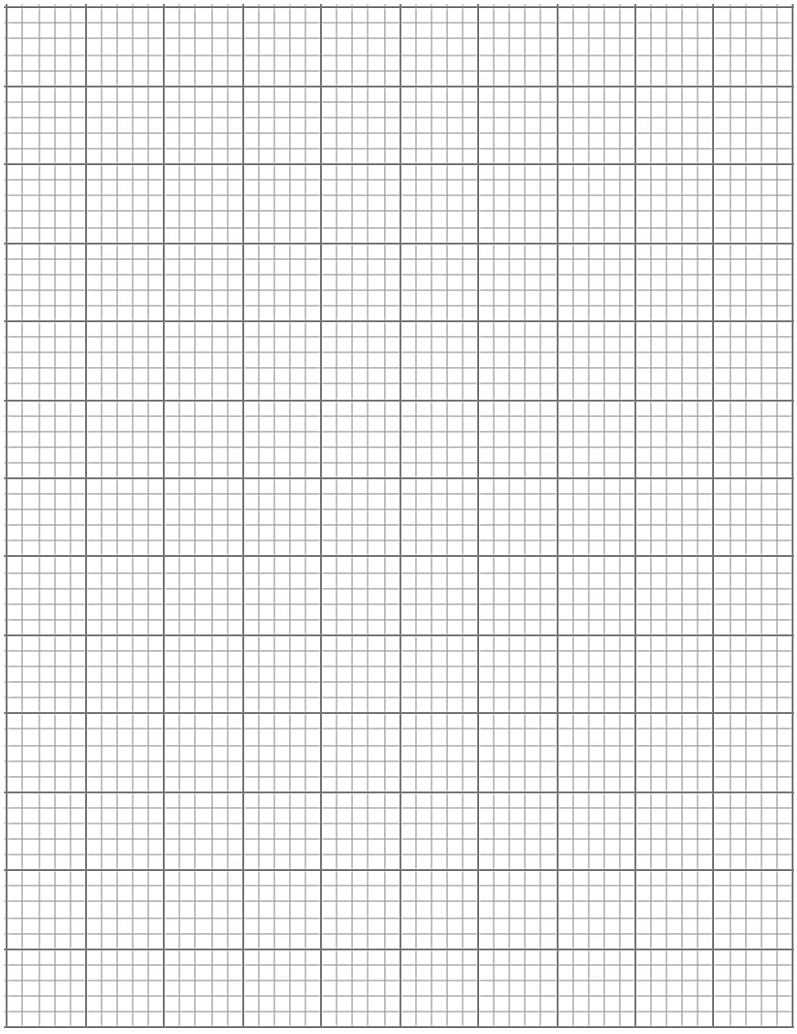
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20. The table below shows values of x and some values of y for the curve $y = 3x^3 + x^2 - 7x$ in the range $-2 \leq x \leq 2$.

(a) Complete the table by filling the missing values of y correct to 1 decimal place. (2 marks)

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
y		2.6	5			–2.9			14

(b) On the grid provided, draw the graph of $y = 3x^3 + x^2 - 7x$ for $-2 \leq x \leq 2$. Use the scale: 2 cm for 1 unit on the x-axis and 2 cm for 5 units on the y-axis. (3 marks)



(c) Use the graph to solve the equation:

(i) $3x^3 + x^2 - 7x - 4 = 0$ (2 marks)

(ii) $3x^3 + x^2 - 10x = 0$ (3 marks)

21.

(a) Fadhili deposited Ksh. 400 000 in an account that paid compound interest on deposits at a rate of 7% p.a. At the end of 3 years, he withdrew all the money from the account.

(i) Calculate the amount that Fadhili withdrew. (2 marks)

(ii) Fadhili invested the withdrawn amount in shares. The value of the shares depreciated at a rate of 1.5% every 6 months. Determine the value of the shares at the end of 2 years correct to 2 decimal places. (3 marks)

(iii) Determine the gain or loss from Fadhili's investments in the 5 years. (1 mark)

(b) Nyambuto invested Ksh. 400 000 in a financial institution that paid compound interest at the rate of 6% per annum. After n years, the amount had accumulated to Ksh. 500 000. Calculate the value of n, correct to the nearest whole number. (4 marks)

22. The probabilities of obtaining scores 1, 2, 3, 4 and 5 using a biased pentagonal spinner were recorded as shown in the following table.

Score	1	2	3	4	5
Probability	k	0.1	0.25	$2k$	0.2

(a) Determine:

(i) the value of k. (2 marks)

(ii) the probability of obtaining a score of 4. (1 mark)

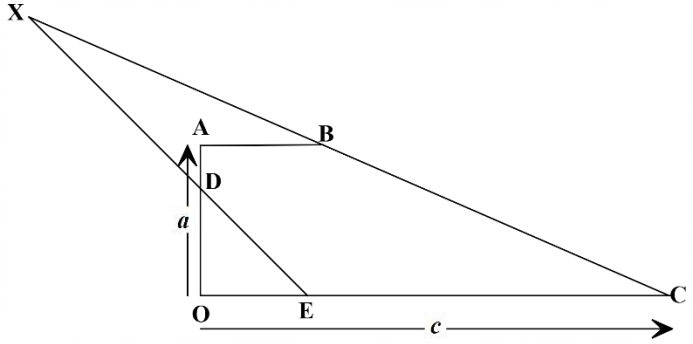
(b) The spinner was span twice.

(i) Work out the probability of obtaining an even number in the first spin and an odd number in the second spin. (4 marks)

(ii) The total score, S, for the two spins was obtained. Determine the probability that $S \geq 9$. (3 marks)

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23. In the figure below, OABC is a trapezium. **AB** is parallel to **OC** and **OC** = 4 **AB**. D is point on **OA** such that OD : DA = 3: 1 and E is a point on **OC** such that OE : EC = 1 : 3.



(a) Given that **OC** = **c** and **OA** = **a**, express in terms of **a** and **c**:

(i) **ED.** (1 mark)

(ii) **CB.** (1 mark)

(b) Line ED and CB produced intersect at X such that **EX** = *h***ED** and **CX** = *k***CB**, where *h* and *k* are scalars.

(i) Express **EX** in terms of **a**, **c** and *h*. (1 mark)

(ii) Express **CX** in terms of **a**, **c** and *k*. (1 mark)

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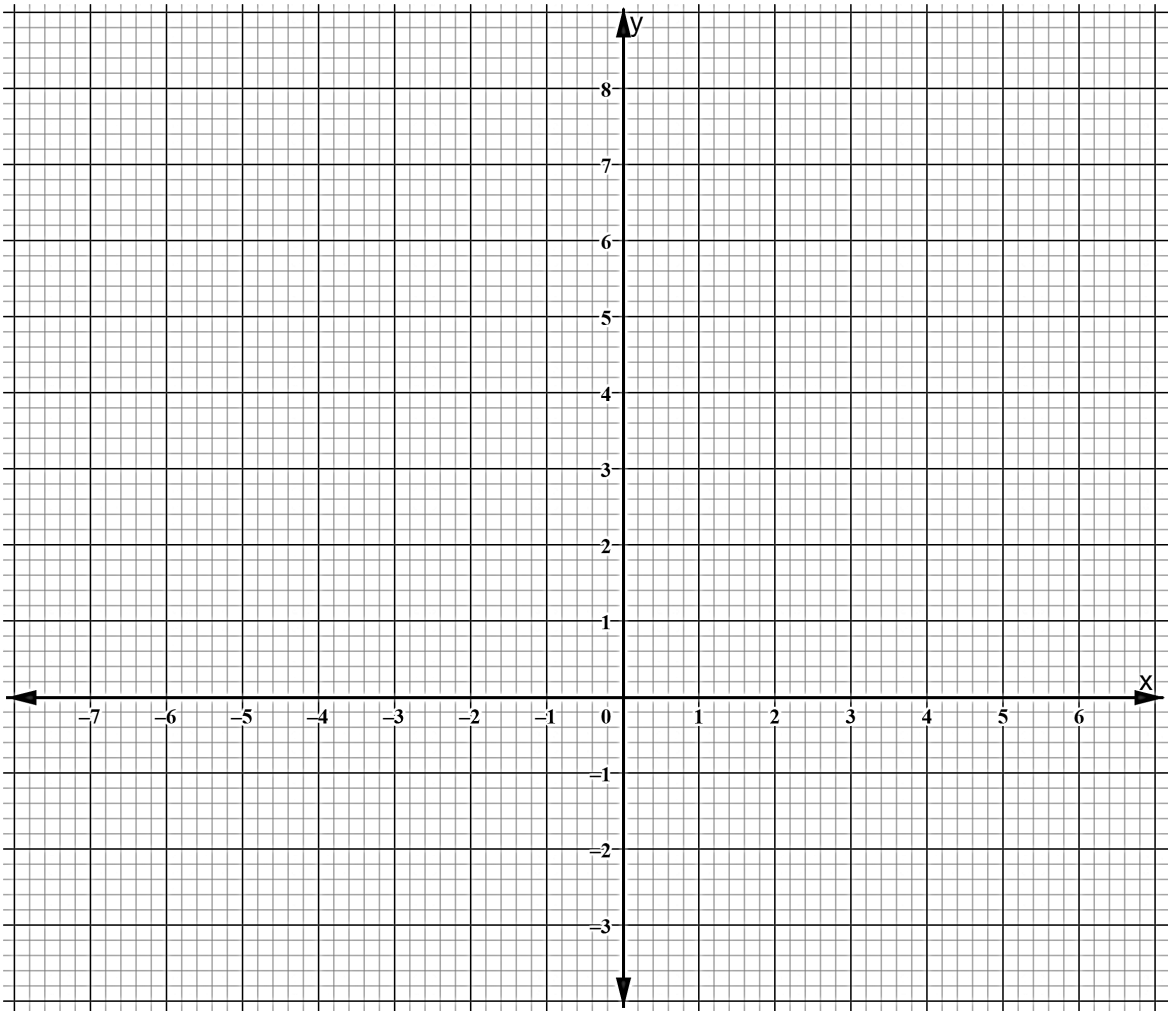
(c) Determine the values of h and k. (5 marks)

(d) Determine the ratio of ED : DX. (1 mark)

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24. PQR is a triangle with vertices $P(5,-1)$, $Q(3,-1)$ and $R(3,-3)$.

(a) On the grid provided, draw triangle PQR. (1 mark)



(b) On the same grid, draw $\Delta P'Q'R'$ the image of ΔPQR under a reflection in the line $y = x$. (3 marks)

(c) $\Delta P''Q''R''$ is the image of $\Delta P'Q'R'$ under a transformation matrix $\mathbf{T} = \begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix}$.

(i) Find the coordinates of $P''Q''R''$. (2 marks)

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(ii) On the same grid, draw triangle $P''Q''R''$. (1 mark)

(d) Determine a single transformation matrix that maps ΔPQR directly to $\Delta P''Q''R''$. (3 marks)

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SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

14/11/2024

1. An arithmetic progression (AP) is given as $600+650+700+750+\ldots$ Determine:

(a) the 30^{th} term of the AP; (2 marks)

$\mathbf{T_n = a+(n-1)d}$

$\mathbf{a = 600,\ n = 30}$

$\mathbf{d = 700-650 = 50}$ ✓

$\mathbf{T_{30} = 600+50(30-1)}$

$\mathbf{= 60+(29\times 50)}$

$\mathbf{= 2050}$ ✓

(b) the sum of the first 30 terms of the AP. (2 marks)

$\mathbf{S_n = \frac{n}{2}[a+l]};\ l = last\ term\ \{30^{th}\ term\}$

$\mathbf{S_{30} = \frac{30}{2}[600+2050]}$ ✓

$\mathbf{= 15\times 2650}$

$\mathbf{= 39750}$ ✓

Alternatively,

$\mathbf{S_n = \frac{n}{2}[2a+(n-1)d]}$

$\mathbf{S_{30} = \frac{30}{2}[(600\times 2)+50(30-1)]}$ ✓

$\mathbf{= 15(1200+1450)}$

$\mathbf{= 39750}$ ✓

2. The quadratic equation $5x^2+kx+20=0$ has only one root. Determine the possible values of k. (2 marks)

For a repeated root the discriminant, $\mathbf{b^2-4ac=0}$ hence $\mathbf{b^2=4ac}$.

$\therefore \mathbf{k^2 = 4\times 5\times 20}$ ✓

$\mathbf{k^2 = 400}$

$\mathbf{k = \pm\sqrt{400} = \pm 20}$ $\mathbf{(20\ or\ -20)}$ ✓

Confidential

3. Without using mathematical tables or a calculator, evaluate $\dfrac{\log 125+\log 64}{\log\sqrt[6]{5}+\log\sqrt[3]{2}}$. (3 marks)

$125=5^3,\ 64=2^6,\ \sqrt[6]{5}=5^{\frac{1}{6}},\ \sqrt[3]{2}=2^{\frac{1}{3}}$

$$\frac{\log\left(5^3\times 2^6\right)}{\log\left(5^{\frac{1}{6}}\times 2^{\frac{1}{3}}\right)}\checkmark=\frac{\log\left(5^1\times 2^2\right)^3}{\log\left(5^1\times 2^2\right)^{\frac{1}{6}}}=\frac{3\log\left(5^1\times 2^2\right)^1}{\frac{1}{6}\log\left(5^1\times 2^2\right)^1}\checkmark=\frac{3}{\frac{1}{6}}=3\times\frac{6}{1}=\mathbf{18}\checkmark$$

Alternatively

$$\frac{\log\left(5^3\times 2^6\right)}{\log\left(5^{\frac{1}{6}}\times 2^{\frac{1}{3}}\right)}\checkmark=\frac{\log 5^3+\log 2^6}{\log 5^{\frac{1}{6}}+\log 2^{\frac{1}{3}}}=\frac{3\log 5+6\log 2}{\frac{1}{6}\log 5+\frac{1}{3}\log 2}\checkmark=\frac{3(\log 5+2\log 2)^1}{\frac{1}{6}(\log 5+2\log 2)^1}=\frac{3}{\frac{1}{6}}=3\times\frac{6}{1}=\mathbf{18}\checkmark$$

4. Make x the subject of the formula $y=\dfrac{a}{b^x}$. (3 marks)

$yb^x=a\Rightarrow \boxed{b^x=\frac{a}{y}}$ ✓

Taking logs on both sides:

$x\log b=\log\left(\frac{a}{b}\right)$ ✓ hence $\boxed{\mathbf{x=\frac{\log\left(\frac{a}{b}\right)}{\log b}}}$ ✓

Alternatively

$yb^x=a\Rightarrow \boxed{b^x=\frac{a}{y}}$ ✓

Taking logs on both sides:

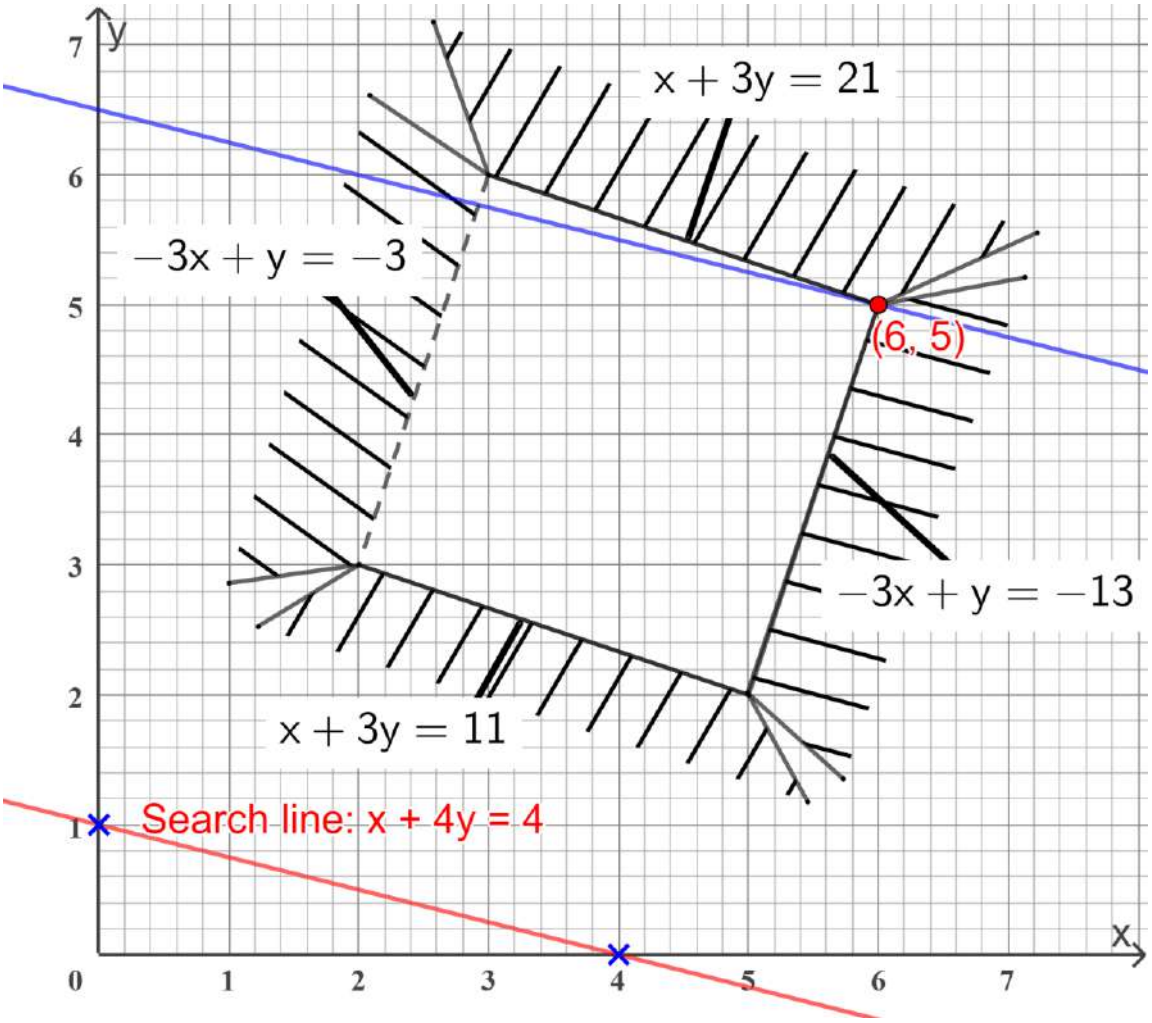
$x\log b=\log\left(\frac{a}{b}\right)$

$x\log b=\log a-\log b$ ✓

$\boxed{\mathbf{x=\frac{\log a-\log b}{\log b}}}$ ✓ or $\boxed{\mathbf{x=\frac{\log a}{\log b}-1}}$ ✓

5. The unshaded region on the Cartesian plane satisfies the inequalities

$x+3y \le 21, -3x+y < -3, -3x+y \ge -13$ and $x+3y \ge 11$.



Find the maximum value of $(x+4y)$ for the integral coordinates $P(x, y)$ lying in the unshaded region.

(3 marks)

Objective function : $\boxed{x+4y=M}$ **Let** $x+4y=4$. **Mark and join points (4, 0) and (0, 1) to obtain the search line.** ✓ **Using a ruler and set square only, produce a parallel line to the search line at the last plausible point (6, 5)** ✓ **before leaving the feasible region.** $M = 6+(4\times 5) = 26$✓

---

6. An aircraft flew due west from point $A(39.64°N, 50°E)$ to $B(39.64°N, 20°W)$. Calculate the distance covered by the aircraft correct to the nearest km. $\left(\text{Take } \pi = \frac{22}{7} \text{ and } R = 6370 \text{ km}\right)$ (3 marks)

Longitude difference, $\theta = 50° + 20° = 70°$✓ (A and B are on either side of the prime meridian)

β = *latitude angle*, R = *Radius of the earth* = $6370\,km$

$$\text{Distance along a parallel of latitude} = \frac{\theta}{360°} \times 2\pi R \cos\beta$$

$$= \frac{70°}{360°} \times 2 \times \frac{22}{7} \times 6370 \times \cos 39.64°$$✓

$$= \frac{70070}{9} \times 0.7701$$

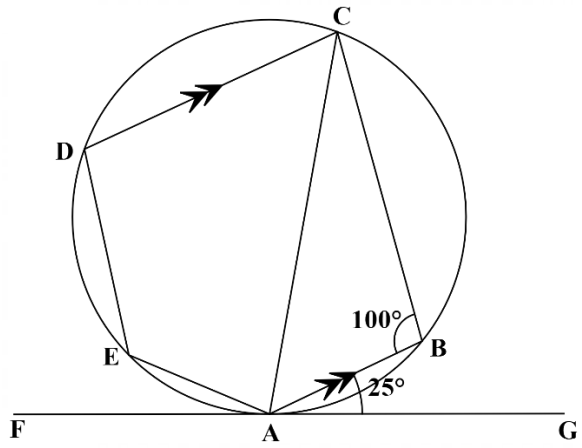
$$= 5995.6563 km$$

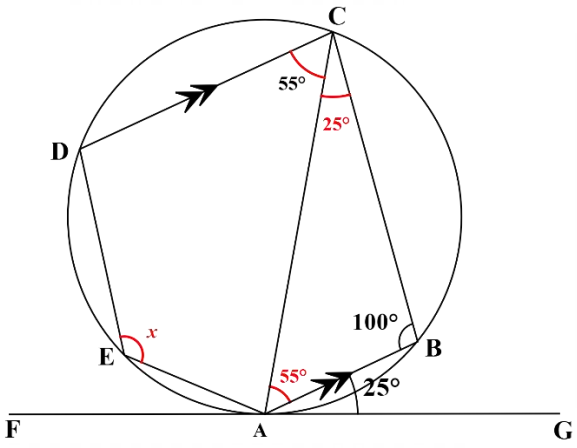
$$\approx \textbf{5996 km}$$✓

✓ Completed

---

7. In the following figure; A, B, C, D and E are points on the circumference of the circle. Line AB is parallel to line DC and line FAG is tangent to the circle at A. Angle GAB = 25° and $\angle ABC = 100°$.





Determine the size of:

(a) $\angle BAC$; (1 mark)

$\angle \mathbf{BCA} = \angle \mathbf{GAB} = 25°$ (by alternate segment//tangent-chord theorem)

$180° - (100° + 25°) = \mathbf{55°}$✓

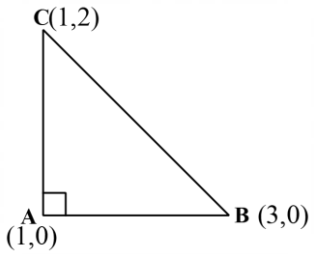
(b) $\angle AED$. (2 marks)

Since AB || CD, AC is a transversal. Thus, $\angle BAC = \angle ACD = 55°$ (*alternate angles*)✓

Opposite angles of cyclic quadrilateral ACDE are supplementary (add up to 180°.)

$\therefore x = 180° - 55° = \mathbf{125°}$✓

8. The triangle ABC with vertices $A(1,0), B(3,0)$ and $C(1,2)$ is transformed by the matrix $\mathbf{T} = \begin{pmatrix} 3k & 1.6 \\ 3k & -0.9 \end{pmatrix}$ onto triangle $A'B'C'$. Given that the area of triangle $A'B'C'$ is 6 square units, determine the value of *k*. (3 marks)



Area of $\triangle ABC = \frac{1}{2} \times 2 \times 2$

$= 2$ *sq.units*

$ASF = \frac{\text{Area of } \triangle A'B'C'}{\text{Area of } \triangle ABC}$

$= \frac{6}{2}$

$= 3$ ✓

$\text{Det } \mathbf{T} = \begin{vmatrix} 3k & 1.6 \\ 3k & -0.9 \end{vmatrix} = 3k(-0.9) - 3k(1.6)$

$= -2.7k - 4.8k$

$= -7.5k$✓

$\boxed{\text{Det } \mathbf{T} = \text{ASF}}$ hence $-7.5k = 3$

$k = \frac{3}{-7.5} = -\frac{\mathbf{1}}{\mathbf{4}}$ or $\mathbf{-0.25}$✓

9. Solve the equation $6\cos^2 x + \sin x = 4$ for $0° \le x \le 180°$, giving the answer correct to 2 decimal places. (3 marks)

$sin^2 x + cos^2 x = 1 \Rightarrow \boxed{\cos^2 x = 1 - sin^2 x}$

$6(1 - \sin x^2) + \sin x = 4$

$6 - 6\sin x^2 + \sin x = 4$

$6\sin x^2 - \sin x - 2 = 0$✓

$6\sin x^2 + 3\sin x - 4\sin x - 2 = 0$

$3\sin x(2\sin x + 1) - 2(2\sin x + 1) = 0$

$(3\sin x - 2)(2\sin x + 1) = 0$✓

$3\sin x - 2 = 0$ or $2\sin x + 1 = 0$

$3\sin x = 2$ $\quad$ $2\sin x = -1$

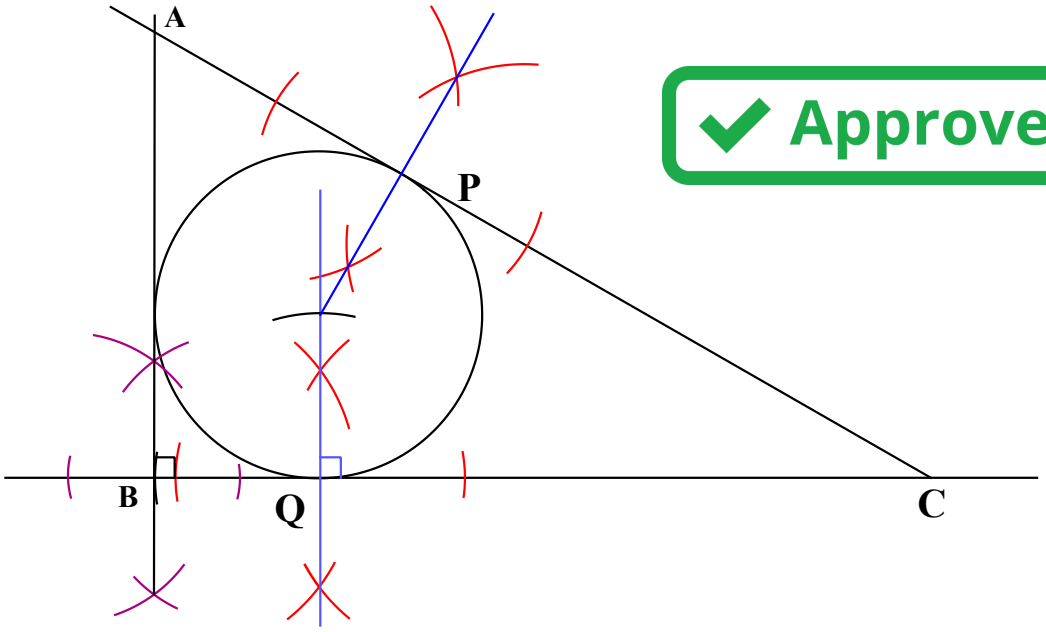
$\sin x = 0.6667$ $\quad$ $\sin x = -0.5$

Since sine is positive in the range $0° \le x \le 180°$;

$x = \sin^{-1} 0.6667$

$= \mathbf{41.81°, 138.19°}$✓

10. The figure below shows a circle. Lines CP and CQ are tangents to the circle at points P and Q respectively.



Approved

The circle is to be inscribed in a triangle ABC. Point B lies on CQ produced and $\angle CBA = 90°$. Use a ruler and a pair of compasses only to:

(a) locate point O, the centre of the circle; (2 marks)

(b) complete triangle ABC. (2 marks)

11. The deviations of the masses of 10 students from an assumed mean are:

$$-10, -5, -2, 1, 4, 5, 7, 8, 9, 13$$

The mass of the heaviest student was 58 kg. Calculate the mean mass of the students. (3 marks)

Let the assumed mean be A. $\therefore\ 58 - A = 13 \Rightarrow A = 58 - 13 = 45$✓

t	−10	−5	−2	1	4	5	7	8	9	13	
f	1	1	1	1	1	1	1	1	1	1	$\sum f = 10$
ft	−10	−5	−2	1	4	5	7	8	9	13	$\sum ft = 30$✓

$$\bar{x} = A + \frac{\sum ft}{\sum f} = 45 + \frac{30}{10} = \mathbf{48}$$✓

---

12. The following table shows part of a monthly income tax rates for a certain year.

Monthly taxable income (Ksh.)	**Tax rate (%)**
0 to 11 180	10
11 181 to 21 714	15
21715 to 32 248	20

In a certain month an employee paid a net tax of Ksh. 2 200 after getting a tax relief of Ksh. 1 280. Calculate the employee's taxable income that month. (3 marks)

Gross tax : $\left(\frac{10}{100} \times 11180\right) + \left(\frac{15}{100} \times 10534\right) + \left(\frac{20}{100} \times y\right) = 2200 + 1280$✓

$$1118 + 1580.10 + 0.2y = 3480$$

$$0.2y = 781.9$$

$$y = \text{Ksh. } 3909.50$$✓

Taxable income = Ksh. $(21714 + 3909.50) =$ **Ksh. 25 623.50**✓

---

13. The equation of a circle is given by $x^2+y^2-3x+4y=0$. Determine:

(a) the coordinates of the centre of the circle; (2 marks)

Compare $x^2+y^2-3x+4y=0$ *with* $\downarrow$

$x^2+y^2-2ax-2by+\boxed{a^2+b^2-r^2}=0$

By comparing : $-2a=-3\Rightarrow a=\dfrac{-3}{-2}=1.5$

$-2b=4 \quad \Rightarrow b=-\dfrac{4}{2}=-2$ ✓

Centre $\boxed{(1.5,-2)}$ ✓

Alternatively

Rearranging: $x^2-3x+y^2+4y=0$

$x^2-3x+\left(\dfrac{-3}{2}\right)^2+y^2+4y+\left(\dfrac{4}{2}\right)^2=\left(\dfrac{-3}{2}\right)^2+\left(\dfrac{4}{2}\right)^2$ ✓

$(x-1.5)^2+(y+2)^2=6.25$

$(x-a)^2+(y-b)^2=r^2$

$-a=-1.5\Rightarrow a=1.5, \quad -b=2\Rightarrow b=-2$

Centre $\boxed{(1.5,-2)}$ ✓

(b) the area of the circle in terms of π. (1 mark)

$r^2=6.25$ hence *area of circle* $=\pi r^2=$ **6.25π sq.units** ✓

or using $a^2+b^2-r^2=c, \quad r^2=a^2+b^2-c$

$r^2=1.5^2+(-2)^2-0$

$=6.25$

Area of circle $=\pi r^2=$ **6.25π sq.units** ✓

14. The position vectors of points A, B and C are such that $\mathbf{OA}=3\mathbf{i}-5\mathbf{j}-4\mathbf{k}, \mathbf{OB}=\mathbf{j}+8\mathbf{k}$ and $\mathbf{OC}=-2\mathbf{i}+5\mathbf{j}+16\mathbf{k}$. Show that the points A, B and C are collinear. (3 marks)

Let **AC** and **AB** be 2 vectors such that $\mathbf{AC}=k\mathbf{AB}$.

$\mathbf{AC}=\mathbf{OC}-\mathbf{OA}=(-2\mathbf{i}+5\mathbf{j}+16\mathbf{k})-(3\mathbf{i}-5\mathbf{j}-4\mathbf{k})$

$=-5\mathbf{i}+10\mathbf{j}+20\mathbf{k}$

$\mathbf{AB}=\mathbf{OB}-\mathbf{OA}=(\mathbf{j}+8\mathbf{k})-(3\mathbf{i}-5\mathbf{j}-4\mathbf{k})$ ✓

$=-3\mathbf{i}+6\mathbf{j}+12\mathbf{k}$

$-5\mathbf{i}+10\mathbf{j}+20\mathbf{k}=k(-3\mathbf{i}+6\mathbf{j}+12\mathbf{k})$

$k=\dfrac{-5}{-3}=\dfrac{10}{6}=\dfrac{20}{12}=\dfrac{5}{3}$ ✓

$\mathbf{AC}=\dfrac{5}{3}\mathbf{AB}$ hence $\mathbf{AC}\parallel\mathbf{A}$ and A is a common point therefore A, B and C are collinear. ✓

Alternatively

$\mathbf{AC}=\begin{pmatrix}-2\\5\\16\end{pmatrix}-\begin{pmatrix}3\\-5\\-4\end{pmatrix}=\begin{pmatrix}-5\\10\\20\end{pmatrix}$

$\mathbf{AB}=\begin{pmatrix}0\\1\\8\end{pmatrix}-\begin{pmatrix}3\\-5\\-4\end{pmatrix}=\begin{pmatrix}-3\\6\\12\end{pmatrix}$ ✓

$\begin{pmatrix}-5\\10\\20\end{pmatrix}=k\begin{pmatrix}-3\\6\\12\end{pmatrix}\Rightarrow k=\dfrac{-5}{-3}=\dfrac{10}{6}=\dfrac{20}{12}=\dfrac{5}{3}$ ✓

$\mathbf{AC}=\frac{5}{3}\mathbf{AB}$ hence $\mathbf{AC}\parallel\mathbf{A}$ and A is a common point therefore A, B and C are collinear. ✓

15. A particle starts from point O and moves in a straight line so that its velocity v ms^{-1} after time t seconds is given by $v=9t^2-18t+10$. Calculate the distance covered by the particle between the time $t=1$ second and $t=2$ seconds. (3 marks)

$S=\int_1^2(9t^2-18t+10)\,dt$

$=\left[\dfrac{9t^{2+1}}{3}-\dfrac{18t^{1+1}}{2}+\dfrac{10t^{0+1}}{0+1}\right]_1^2$ ✓

$=\left[3t^3-9t^2+10t\right]_1^2$

$=\left[3(2^3)-9(2^2)+10(2)\right]-\left[3(1^3)-9(1^2)+10(1)\right]$ ✓

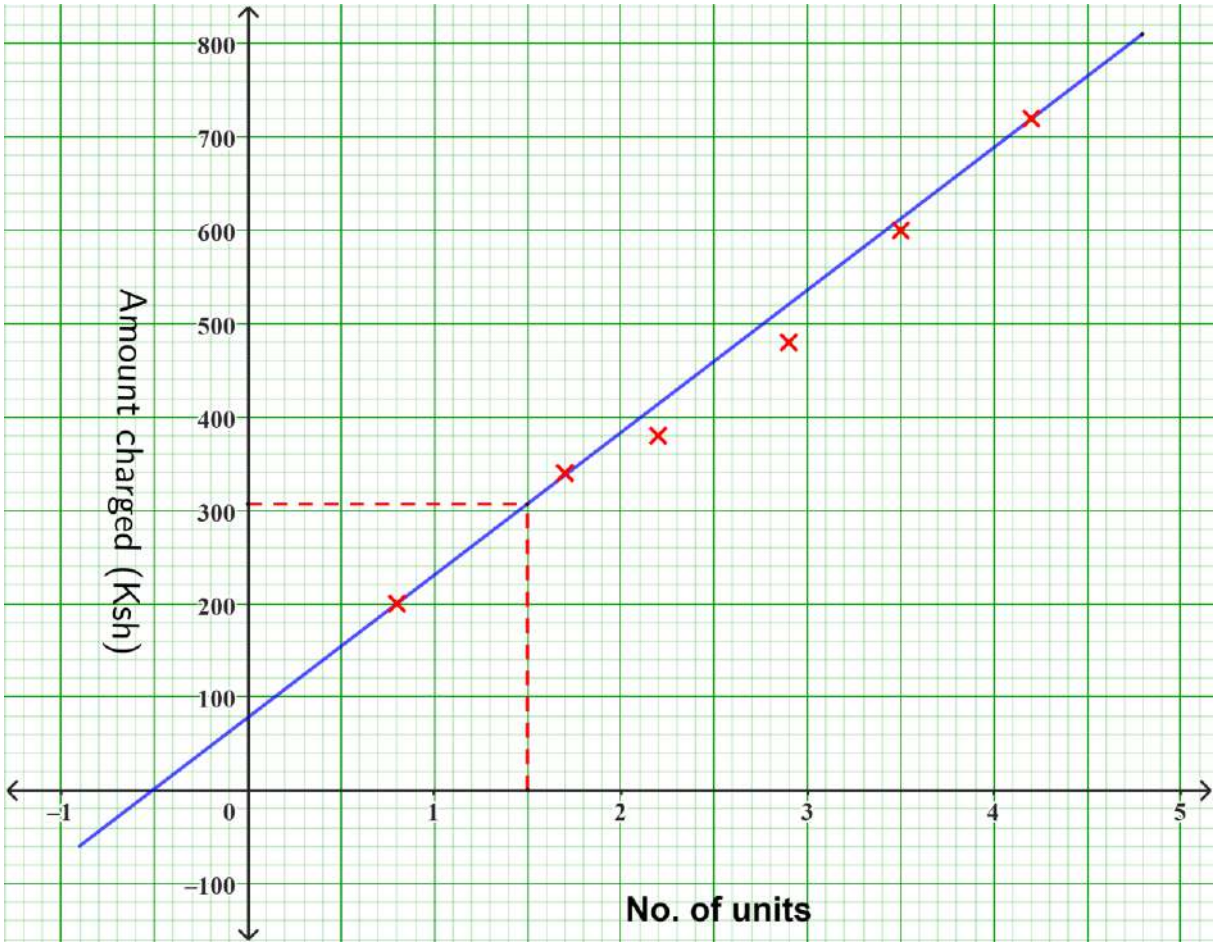
$=8-4$

$=$ **4 metres** ✓

16. The following table shows the number of units (U) of water consumed by 6 households in a month. The corresponding amount (A) charged is also given.

No. of units (U)	0.8	1.7	2.2	2.9	3.5	4.2
Amount (A) charged in Ksh.	200	340	380	480	600	720

(a) Using the scale 2 cm to represent 1 unit on the *x*-axis and 1 cm to represent Ksh. 100 on the *y*-axis, draw the line of best fit for the data on the grid provided. (3 marks)



(b) Estimate the cost of 1.5 units of water. (1 mark)

≈ **Ksh. 310**✓

4 850983 348776

Approved

SECTION II (50 marks)

*Answer only **five** questions in this section in the spaces provided.*

17. A poultry dealer has to types of chicken feeds: type A and type B. He sells 1 kg of type A at Ksh. 45 and 1 kg type B at Ksh. 30. He makes a profit of 20% per kg of type A feed sold and 25% per kg of typer B feed sold. He also sales mixtures of type A and type B feeds.

(a) Determine the amount of profit made by the dealer for selling 1 kg of:

(i) type A feed. (1 mark)

$\text{Profit} = \frac{20}{120} \times 45 = \underline{\underline{\textbf{Ksh. 7.50}}}$ ✓ OR $\text{Profit} = 45 - \left(\frac{100 \times 45}{120}\right) = \underline{\underline{\textbf{Ksh. 7.50}}}$ ✓

(ii) type B feed. (1 mark)

$\text{Profit} = \frac{25}{125} \times 30 = \underline{\underline{\textbf{Ksh. 6}}}$ ✓ OR $\text{Profit} = 30 - \left(\frac{100 \times 30}{125}\right) = \underline{\underline{\textbf{Ksh. 6}}}$ ✓

(b) Type A and type B feeds were mixed in the ratio $3:7$. Calculate:

(i) the selling price of 1 kg of the mixture; (2 marks)

$$\underset{\text{(per kg of mixture)}}{\text{Selling price}} = \frac{(3 \times 45) + (7 \times 30)}{3 + 7} \checkmark = \frac{345}{10}$$

$$= \underline{\underline{\textbf{Ksh. 34.50}}} \checkmark$$

(ii) the profit made by the dealer in selling 50 kg of the mixture. (2 marks)

$$\text{Profit in selling 1 kg of mixture} = \frac{(3 \times 7.50) + (7 \times 6)}{10} = \text{Ksh. } 6.45 \checkmark$$

$$\text{Proft in selling 50 kg of mixture} = 50 \times 6.45 = \underline{\underline{\textbf{Ksh. 322.50}}} \checkmark$$

(c) The dealer made a profit of Ksh. 1 387.50 for the sale of 200 kg of a different mixture of type A and type B feeds. Determine the ratio of type A feed to that of type B feed in the mixture. (4 marks)

Let the ratio of A:B in the mixture be $x : y$.

$$200\left(\frac{7.50x + 6y}{x + y}\right) = 1387.50 \checkmark$$

Dividing both sides by 200...

$$\frac{7.50x + 6y}{x + y} = \frac{111}{16}$$

$$16(7.50x + 6y) = 111(x + y) \checkmark$$

$$120x + 96y = 111x + 111y$$

$$9x = 15y \checkmark$$

Dividing both sides by $9y$...

$$\frac{x}{y} = \frac{5}{3}$$

$$\therefore \ \underline{\underline{\textbf{A : B = 5 : 3}}} \checkmark$$

18.

(a) A quantity P is partly constant and partly varies as the square root of a quantity Q. Given that P = 20 when Q = 4 and that P = 60 when Q = 100, find Q when P = 22. (4 marks)

$P = C + k\sqrt{Q}$

$20 = C + k\sqrt{4} \Rightarrow 20 = C + 2k \quad ...(i)$

$60 = C + k\sqrt{100} \Rightarrow 60 = C + 10k \quad ...(ii)$ ✓

$(ii) - (i): \; 8k = 40$

$k = 5$

Using (i), $C = 20 - (2 \times 5) = 10$

$\therefore \boxed{P = 10 + 5\sqrt{Q}}$ ✓

When $P = 22$; $5\sqrt{Q} = 22 - 10$ ✓

$5\sqrt{Q} = 12$

$\sqrt{Q} = 2.4$

$Q = 2.4^2$

$= \mathbf{5.76}$ ✓

(b) Three quantities, T, U and V are such that T varies directly as the square of $(10 - U)$ and inversely as the cube root of V. When T = 12, U = 4 and V = 8.

(i) Determine the equation connecting T, U and V. (3 marks)

$T \alpha \dfrac{(10-U)^2}{\sqrt[3]{V}}$

$\therefore T = \dfrac{k(10-U)^2}{\sqrt[3]{V}}$ ✓

Given T = 12 when U = 4 and V = 8;

$12 = \dfrac{k(10-4)^2}{\sqrt[3]{8}}$

$12 = \dfrac{k \times 6^2}{2}$

$36k = 24$

$k = \dfrac{24}{36} = \dfrac{2}{3}$ ✓

$\therefore$ the equation: $\boxed{\mathbf{T = \dfrac{\frac{2}{3}(10-U)^2}{\sqrt[3]{V}}}}$ ✓ or $\boxed{\mathbf{T = \dfrac{2(10-U)^2}{3\sqrt[3]{V}}}}$ ✓

(ii) Find U when $T = 5\dfrac{2}{5}$ and $V = 15\dfrac{5}{8}$. (3 marks)

$\dfrac{27}{5} = \dfrac{2(10-U)^2}{3 \times \sqrt[3]{15.625}}$

$27 \times 3 \times \sqrt[3]{15.625} = 5 \times 2(10-U)^2$

$27 \times 3 \times 2.5 = 10(10-U)^2$ ✓

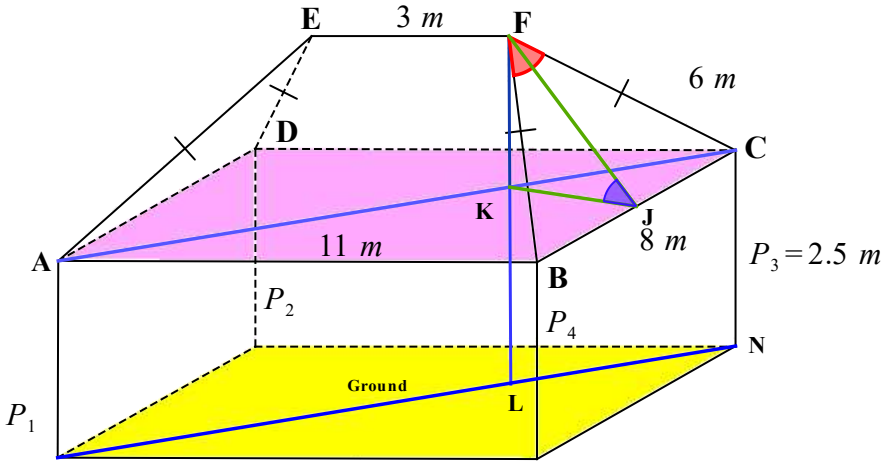
$\dfrac{27 \times 3 \times 2.5}{10} = \cancel{10}^{1}(10-U)^2 \times \dfrac{1}{\cancel{10}^{1}}$

$\sqrt{20.25} = \sqrt{(10-U)^2}$ ✓

$\pm 4.5 = 10 - U$

$\therefore U = 10 - 4.5 = \mathbf{5.5}$ ✓ Or $U = 10 + 4.5 = \mathbf{14.5}$ ✓

19. The following figure shows a tent erected on a level ground. The roof ABCDEF of the tent is supported by four vertical posts P_1, P_2, P_3 and P_4 each of height 2.5 m. The ridge EF = 3 m is centrally placed. Further, AB = 11 m, BC = 8 m and FB = FC = ED = EA = 6 m.



Calculate:

(a) the length of the projection of FC on the ground correct to 4 significant figures. (3 marks)

$KJ = \frac{11-3}{2} = 4m$ ✓ | $BJ = JC = \frac{8}{2} = 4m$

Plane ABCD = Rectangular part of the ground formed by the supporting posts then translated.

The projection of FC on the ground is LN.

But LN = KC

$KC = \sqrt{KJ^2 + JC^2}$

$= \sqrt{4^2 + 4^2}$ ✓

$=$ **5.657 m** ✓

(b) the height of the ridge EF above the ground. (3 marks)

Required length is FL.

$FL = FK + KL$

$= FK + 2.5m$

Using $\triangle FKC$;

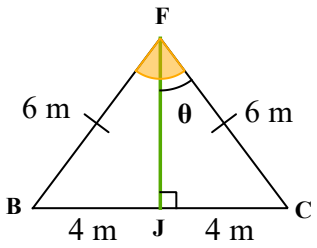
$FK = \sqrt{FC^2 - KC^2} = \sqrt{6^2 - 32}$ ✓

$= 2m$ ✓

$FL = 2m + 2.5m$

$=$ **4.5 m** ✓

(c) the angle between edge FB and edge FC. (2 marks)



Required angle is $\angle BFC$.

$\sin\theta = \frac{4}{6} \Rightarrow \theta = \sin^{-1} 0.6667$

$= 41.81°$ ✓

$\angle BFC = 2 \times 41.81°$

$=$ **83.62°** ✓

Alternatively, using cosine rule;

$8^2 = 6^2 + 6^2 - (2 \times 6 \times 6 \cos F)$

$\angle F = \cos^{-1}\left(\frac{6^2 + 6^2 - 8^2}{2 \times 6 \times 6}\right)$ ✓

$= \cos^{-1} 0.1111$

$=$ **83.62°** ✓

(d) the angle between the plane FBC and the ground. (2 marks)

Angle between the plane FBC and the ground = angle between the plane FBC and pane ABCD

= angle between FJ and its projection JK on ABCD

Using triangle FKJ; $\tan J = \frac{FK}{KJ} \Rightarrow J = \tan^{-1}\left(\frac{FK}{KJ}\right) = \tan^{-1}\left(\frac{2}{4}\right)$ ✓

$= \tan^{-1} 0.5$

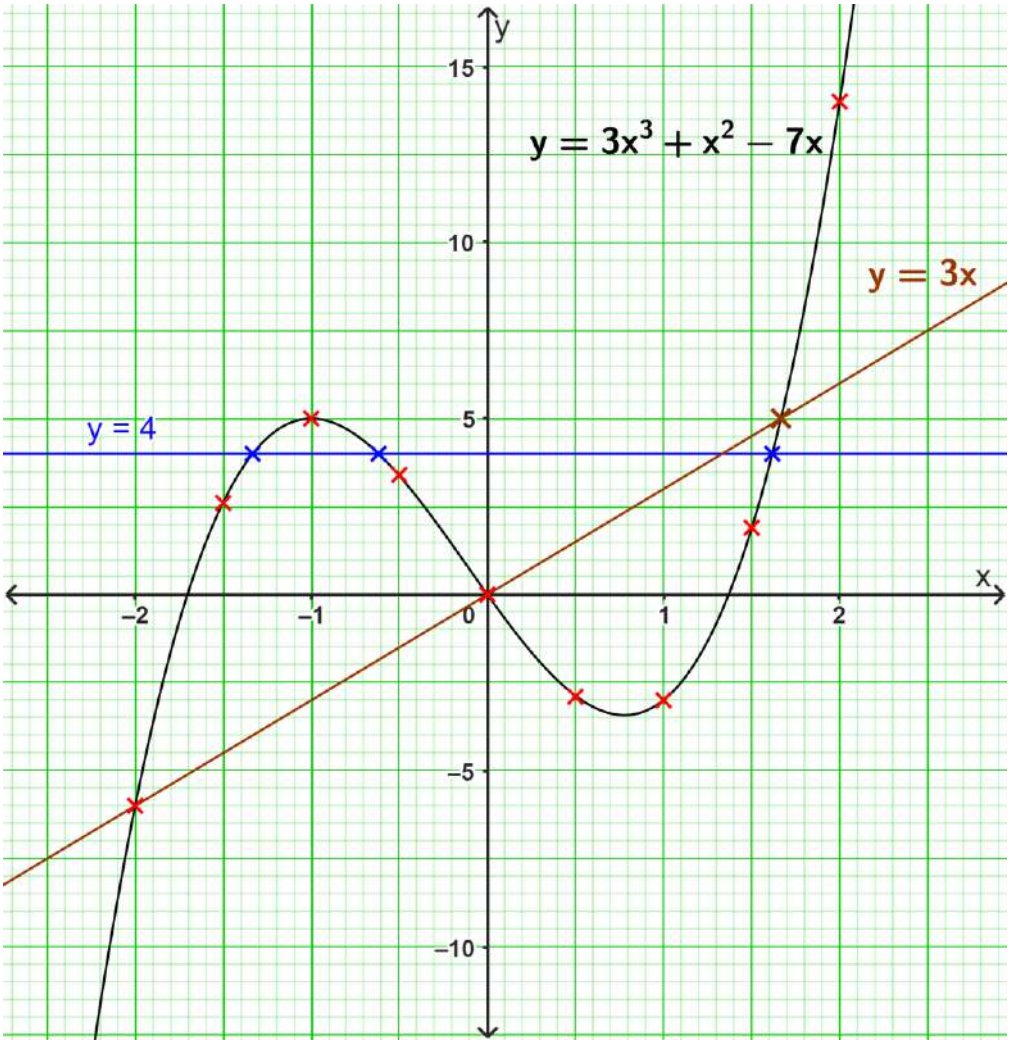
$=$ **26.57°** ✓

20. The table below shows values of x and some values of y for the curve $y = 3x^3 + x^2 - 7x$ in the range $-2 \le x \le 2$.

(a) Complete the table by filling the missing values of y correct to 1 decimal place. (2 marks)

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	**−6**	2.6	5	**3.4**	**0**	−2.9	**−3**	**1.9**	14

(b) On the grid provided, draw the graph of $y = 3x^3 + x^2 - 7x$ for $-2 \le x \le 2$. Use the scale: 2 cm for 1 unit on the x-axis and 2 cm for 5 units on the y-axis. (3 marks)



(c) Use the graph to solve the equation:

(i) $3x^3 + x^2 - 7x - 4 = 0$ (2 marks)

$y = 3x^3 + x^2 - 7x$

$0 = -4 + 3x^3 + x^2 - 7x$ $\}-$

$y = 4$ ✓

$x = -1.3,\ -0.6,\ 1.6$ ✓

(ii) $3x^3 + x^2 - 10x = 0$ (3 marks)

$y = -7x + x^2 + 3x^3$

$0 = -10x + x^2 + 3x^3$ $\}-$

$y = 3x$ ✓

x	0	−2
y	0	−6

$x = -2,\ 0,\ 1.7$ ✓

21.

(a) Fadhili deposited Ksh. 400 000 in an account that paid compound interest on deposits at a rate of 7% p.a. At the end of 3 years, he withdrew all the money from the account.

(i) Calculate the amount that Fadhili withdrew. (2 marks)

$$\text{Accumulated amount} = 400,000\left(1+\frac{7}{100}\right)^3 \checkmark$$
$$= 400,000 \times 1.225043$$
$$= \text{Ksh. } 490{,}017.20 \qquad \therefore \text{Amount he withdrew is } \underline{\underline{\textbf{Ksh. 490, 017}}} \checkmark$$

(ii) Fadhili invested the withdrawn amount in shares. The value of the shares depreciated at a rate of 1.5% every 6 months. Determine the value of the shares at the end of 2 years correct to 2 decimal places. (3 marks)

$$A = P\left(1-\frac{r}{100}\right)^n ;\ n = 2\times 2 = 4 \checkmark$$
$$\text{Amount} = 490{,}017.20\left(1-\frac{1.5}{100}\right)^4 \checkmark$$
$$= 490{,}017.20 \times 0.94133655$$
$$= \underline{\underline{\textbf{Ksh. 461,217.10}}} \checkmark$$

(iii) Determine the gain or loss from Fadhili's investments in the 5 years. (1 mark)

Negative if loss, positive if gain.

$$\text{Gain/loss} = \text{Amount at the end of 5 years} - \text{Initial amount}$$
$$= \text{Ksh. } (461{,}217.10 - 400{,}000)$$
$$= \underline{\underline{\textbf{Ksh. 61,217.10 (gain)}}} \checkmark$$

(b) Nyambuto invested Ksh. 400 000 in a financial institution that paid compound interest at the rate of 6% per annum. After n years, the amount had accumulated to Ksh. 500 000. Calculate the value of n, correct to the nearest whole number. (4 marks)

From $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow \left(1+\frac{r}{100}\right)^n = \frac{A}{P}$

Taking logs on both sides;

$$n\log\left(1+\frac{r}{100}\right) = \log A - \log P$$
$$\therefore n = \frac{\log A - \log P}{\log\left(1+\frac{r}{100}\right)} \checkmark$$

$$n = \frac{\log 500000 - \log 400000}{\log\left(1+\frac{6}{100}\right)} \checkmark$$
$$= \frac{5.69897 - 5.60206}{0.0253059} \checkmark$$
$$= 3.8295$$
$$\approx \underline{\underline{\textbf{4 years}}} \checkmark$$



22. The probabilities of obtaining scores 1, 2, 3, 4 and 5 using a biased pentagonal spinner were recorded as shown in the following table.

Score	1	2	3	4	5
Probability	k	0.1	0.25	$2k$	0.2

(a) Determine:

(i) the value of k. (2 marks)

$$k+0.1+0.25+2k+0.2=1\checkmark$$

$$3k+0.55=1$$

$$3k=1-0.55$$

$$3k=0.45 \Rightarrow k=\frac{0.45}{3}=0.15\checkmark$$

(ii) the probability of obtaining a score of 4. (1 mark)

$$P(score=4)=2k$$

$$=2\times 0.15=0.3\checkmark$$

(b) The spinner was span twice.

(i) Work out the probability of obtaining an even number in the first spin and an odd number in the second spin. (4 marks)

		First spin				
		1	**2**	**3**	**4**	**5**
Second spin	**1**	(1,1)	**(2,1)**	(3,1)	**(4,1)**	(5,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)
	3	(1,3)	**(2,3)**	(3,3)	**(4,3)**	(5,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)
	5	(1,5)	**(2,5)**	(3,5)	**(4,5)**	(5,5)

✓✓ 2 marks

Probability space = 25
Favourable outcomes = 6 } ✓

$$P(even\ and\ odd)=\frac{6}{25} \text{ or } 0.24\checkmark$$

Alternatively:

$$P(2,1)\ \textbf{OR}\ P(2,3)\ \textbf{OR}\ P(2,5)\ \textbf{OR}\ P(4,1)\ \textbf{OR}\ P(4,3)\ \textbf{OR}\ P(4,5)\checkmark$$

$$P(2,1)+P(2,3)+P(2,5)+P(4,1)+P(4,3)+P(4,5)$$

$$(0.1\times 0.15)+(0.1\times 0.25)+(0.1\times 0.2)+(0.3\times 0.15)+(0.3\times 0.25)+(0.3\times 0.2)\checkmark$$

$$0.015+0.025+0.02+0.045+0.075+0.06\checkmark$$

$$\textbf{0.24}\checkmark$$

(ii) The total score, **S**, for the two spins was obtained. Determine the probability that $S\geq 9$. (3 marks)

$$P(4,5)\ \textbf{OR}\ P(5,4)\ \textbf{OR}\ P(5,5)\checkmark$$

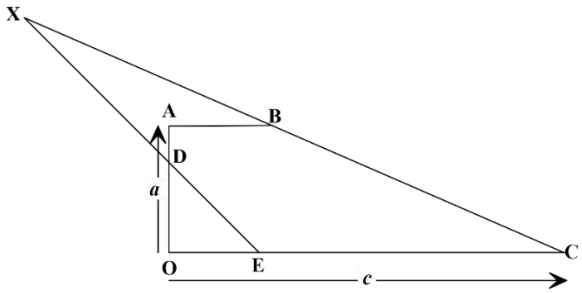
$$P(4,5)+\ P(5,4)\ +\ P(5,5)$$

$$(0.3\times 0.2)+(0.2\times 0.3)+(0.2\times 0.2)\checkmark$$

$$0.06+0.06+0.04$$

$$\underline{\underline{\textbf{0.16}\checkmark}}$$

23. In the figure below, OABC is a trapezium. **AB** is parallel to **OC** and **OC** = 4 **AB**. D is point on **OA** such that OD : DA = 3: 1 and E is a point on **OC** such that OE : EC = 1 : 3.



(a) Given that **OC** = **c** and **OA** = **a**, express in terms of **a** and **c**:

(i) **ED.** (1 mark)

$$\begin{aligned}\mathbf{ED} &= \mathbf{OD} - \mathbf{OE}\\ &= \tfrac{3}{4}\mathbf{OA} - \tfrac{1}{4}\mathbf{OC}\\ &= \boxed{\tfrac{3}{4}\mathbf{a} - \tfrac{1}{4}\mathbf{c}} \text{ or } \boxed{\tfrac{1}{4}(3\mathbf{a} - \mathbf{c})} \checkmark\end{aligned}$$

(ii) **CB.** (1 mark)

$$\begin{aligned}\mathbf{CB} &= \mathbf{OB} - \mathbf{OC}\\ &= \mathbf{OA} + \mathbf{AB} - \mathbf{OC}\\ &= \mathbf{OA} + \tfrac{1}{4}\mathbf{OC} - \mathbf{OC}\\ &= \mathbf{a} + \tfrac{1}{4}\mathbf{c} - \mathbf{c}\\ &= \boxed{\mathbf{a} - \tfrac{3}{4}\mathbf{c}} \text{ or } \boxed{\tfrac{1}{4}(4\mathbf{a} - 3\mathbf{c})} \checkmark\end{aligned}$$

(b) Line ED and CB produced intersect at X such that **EX** = h**ED** and **CX** = k**CB**, where h and k are scalars.

(i) Express **EX** in terms of **a**, **c** and h. (1 mark)

$$\mathbf{EX} = h\left(\frac{3}{4}\mathbf{a} - \frac{1}{4}\mathbf{c}\right) = \boxed{\frac{3}{4}h\mathbf{a} - \frac{1}{4}h\mathbf{c}} \text{ or } \boxed{\frac{1}{4}h(3\mathbf{a} - \mathbf{c})} \checkmark$$

(ii) Express **CX** in terms of **a**, **c** and k. (1 mark)

$$\mathbf{CX} = k\left(\mathbf{a} - \frac{3}{4}\mathbf{c}\right) = \boxed{k\mathbf{a} - \frac{3}{4}k\mathbf{c}} \text{ or } \boxed{\frac{1}{4}k(4\mathbf{a} - 3\mathbf{c})} \checkmark$$

(c) Determine the values of h and k. (5 marks)

Expressing **OX** in 2 ways:

$$\begin{aligned}\mathbf{OX} &= \mathbf{OE} + \mathbf{EX}\\ &= \tfrac{1}{4}\mathbf{c} + \tfrac{3}{4}h\mathbf{a} - \tfrac{1}{4}h\mathbf{c}\\ &= \tfrac{3}{4}h\mathbf{a} + \left(\tfrac{1}{4} - \tfrac{1}{4}h\right)\mathbf{c} \ldots\ldots (i) \checkmark\end{aligned}$$

$$\begin{aligned}\mathbf{OX} &= \mathbf{OC} + \mathbf{CX}\\ &= \mathbf{c} + k\mathbf{a} - \tfrac{3}{4}k\mathbf{c}\\ &= k\mathbf{a} + \left(1 - \tfrac{3}{4}k\right)\mathbf{c} \ldots\ldots (ii) \checkmark\end{aligned}$$

$\boxed{k = \frac{3}{4}h}$ and $\boxed{1 - \frac{3}{4}k = \frac{1}{4} - \frac{1}{4}h}$

Substituting for k in $\uparrow$

$$1 - \frac{3}{4}\left(\frac{3}{4}h\right) = \frac{1}{4} - \frac{1}{4}h \checkmark$$

$$1 - \frac{9}{16}h = \frac{1}{4} - \frac{1}{4}h$$

$$\frac{5}{16}h = \frac{3}{4} \Rightarrow h = \frac{3}{4} \times \frac{16}{5} = \mathbf{\frac{12}{5}} \checkmark$$

$$k = \frac{3}{4} \times \frac{12}{5} = \mathbf{\frac{9}{5}} \checkmark$$

Alternatively, express **CX** or **EX** in 2 ways…

$$\begin{cases}\mathbf{CX} = \mathbf{CE} + \mathbf{EX} \ldots\ldots\ldots\ldots\ldots (i)\\ \quad = -\tfrac{1}{4}(3 + h)\mathbf{c} + \tfrac{3}{4}h\mathbf{a}\\ \mathbf{CX} = k\mathbf{a} - \tfrac{3}{4}k\mathbf{c} \ldots\ldots\ldots\ldots\ldots (ii)\end{cases}$$

$$\begin{cases}\mathbf{EX} = \mathbf{EC} + \mathbf{CX}\\ \quad = k\mathbf{a} + \left(\tfrac{3}{4} - \tfrac{3}{4}k\right)\mathbf{c} \ldots\ldots (i)\\ \mathbf{EX} = \tfrac{3}{4}h\mathbf{a} - \tfrac{1}{4}h\mathbf{c} \ldots\ldots\ldots\ldots (ii)\end{cases}$$

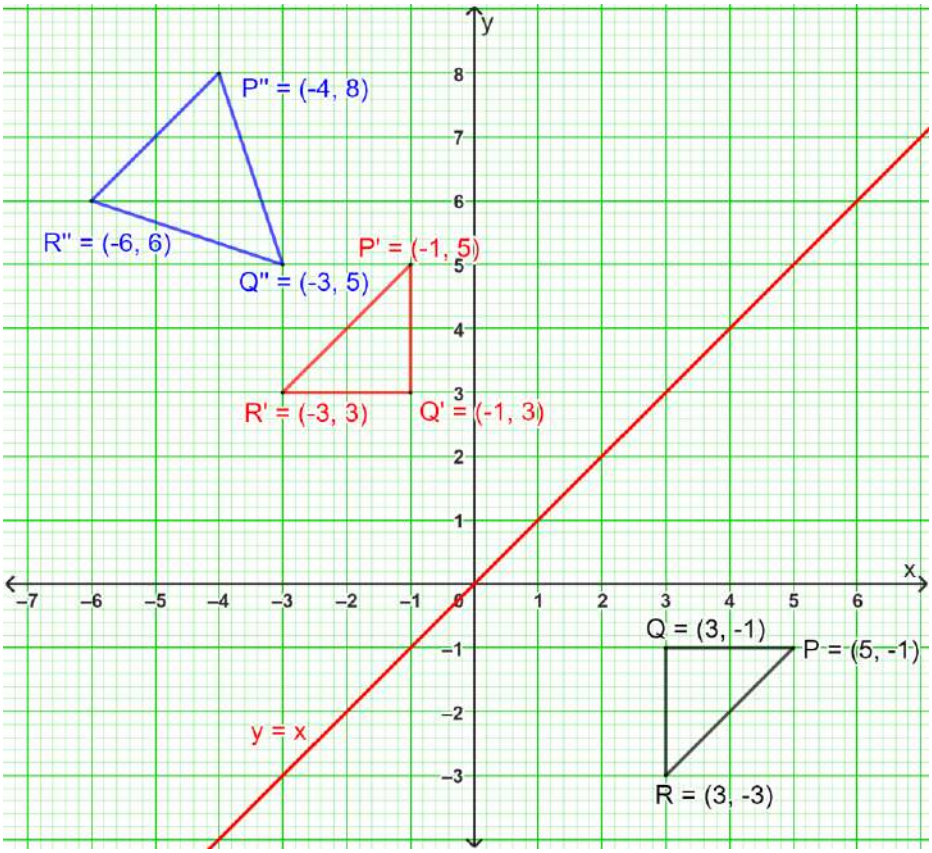
(d) Determine the ratio ED : DX. (1 mark)

$$\mathbf{DX} = \frac{7}{20}(3\mathbf{a} - \mathbf{c}) \text{ and } \mathbf{ED} = \frac{1}{4}(3\mathbf{a} - \mathbf{c}) \Rightarrow \frac{\text{ED}}{\text{DX}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$$

$$\therefore \text{ED} : \text{DX} = \mathbf{5 : 7} \checkmark$$

24. PQR is a triangle with vertices $P(5,-1)$, $Q(3,-1)$ and $R(3,-3)$.

(a) On the grid provided, draw triangle PQR. (1 mark)



(b) On the same grid, draw $\Delta P'Q'R'$ the image of ΔPQR under a reflection in the line $y = x$. (3 marks)

$X(a,b) \xrightarrow[y=x]{\text{Reflect}} X'(b,a)$, use the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or construct.

$$\begin{matrix} & P & Q & R \end{matrix}$$
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 5 & 3 & 3 \\ -1 & -1 & -3 \end{pmatrix} = \begin{pmatrix} (0\times5)+(1\times-1) & (0\times3)+(1\times-1) & (0\times3)+(1\times-3) \\ (1\times5)+(0\times-1) & (1\times3)+(0\times-1) & (1\times3)+(0\times-3) \end{pmatrix} \checkmark = \begin{matrix} P' & Q' & R' \end{matrix}\begin{pmatrix} -1 & -1 & -3 \\ 5 & 3 & 3 \end{pmatrix}$$

$\therefore$ $\mathbf{P'(-1,5), Q'(-1,3), R'(-3,3)}$ ✓

(c) $\Delta P''Q''R''$ is the image of $\Delta P'Q'R'$ under a transformation matrix $\mathbf{T} = \begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix}$.

(i) Find the coordinates of $P''Q''R''$. (2 marks)

$$\begin{matrix} & P' & Q' & R' \end{matrix}$$
$$\begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix}\begin{pmatrix} -1 & -1 & -3 \\ 5 & 3 & 3 \end{pmatrix} = \begin{pmatrix} (1.5\times-1)+(-0.5\times5) & (1.5\times-1)+(-0.5\times3) & (1.5\times-3)+(-0.5\times3) \\ (-0.5\times-1)+(1.5\times5) & (-0.5\times-2)+(1.5\times3) & (-0.5\times-3)+(1.5\times3) \end{pmatrix} \checkmark$$
$$= \begin{matrix} P'' & Q'' & R'' \end{matrix}\begin{pmatrix} -4 & -3 & -6 \\ 8 & 5 & 6 \end{pmatrix}$$

$\therefore$ $\mathbf{P''(-4,8), Q''(-3,5), R''(-6,6)}$ ✓

(ii) On the same grid, draw triangle $P''Q''R''$. (1 mark)

(d) Determine a single transformation matrix that maps ΔPQR directly to $\Delta P''Q''R''$. (3 marks)

$$\begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \checkmark = \begin{pmatrix} (1.5\times0)+(-0.5\times1) & (1.5\times1)+(-0.5\times0) \\ (-0.5\times0)+(1.5\times1) & (-0.5\times1)+(1.5\times0) \end{pmatrix} \checkmark = \begin{pmatrix} \mathbf{-0.5} & \mathbf{1.5} \\ \mathbf{1.5} & \mathbf{-0.5} \end{pmatrix} \checkmark$$